

Corso di Analisi Matematica T-B
Corso di Laurea in Ingegneria Meccanica
Anno Accademico 2012/13

Esercizi

A) Calcolare i seguenti integrali:

1. $\int_0^{1/3} e^{3x+1} dx$

2. $\int_0^{1/3} \arctan(3x) dx$

3. $\int_1^e \frac{\cos(\pi \log x)}{x} dx$

4. $\int_0^2 x^2 \sin(x^3) dx$

5. $\int_0^{\pi/2} \frac{\cos x}{1 + \sin^2 x} dx$

6. $\int_{-\log 2}^0 e^{2x} \exp(e^{2x} + 1) dx$

7. $\int_{\pi/4}^{\pi/2} (x + 2) \cos x dx$

8. $\int_1^2 (x^2 - 2x)e^{2x} dx$

9. $\int_0^{2/5} (5x - 2)^2 e^{-5x} dx$

10. $\int_{-2}^0 2x \log(x + 5) dx$

11. $\int_0^2 x^2 \arctan x dx$

12. $\int_0^1 x^2 \arctan(x^3) dx$

13. $\int_0^2 x^5 e^{x^3} dx$

14. $\int_1^2 \frac{x^2 + 1}{x + 1} dx$

15. $\int_1^3 \frac{x + 1}{3x^2 + 2x} dx$

16. $\int_0^2 \frac{1}{x^2 + 4x + 3} dx$

17. $\int_1^2 \frac{x + 5}{x^2 + 4x + 4} dx$

18. $\int_0^2 \frac{x^2 + 1}{3x^2 + 5} dx$

19. $\int_0^2 \frac{x^3 + x^2 + 1}{4x^2 + 4x + 1} dx$

20. $\int_2^3 \frac{1}{x^3 - x} dx$

21. $\int_0^3 \frac{2x}{x^2 - x + 2} dx$

22. $\int_0^1 \frac{e^x + 1}{2e^x + 3} dx$

23. $\int_0^1 \frac{e^x + 2}{e^{-x} + 3} dx$

24. $\int_0^{1/2} \frac{1}{e^{2x} - 2e^x} dx$

25. $\int_0^1 \frac{e^x}{e^{2x} + 1} dx$

26. $\int_{\pi/4}^{\pi/2} (\sin^4 x + \cos^4 x) dx$

27. $\int_{\pi/6}^{\pi/2} \frac{\cos x}{2 \sin^2 x + \cos^2 x - 5} dx$

$$28. \int_0^{\pi/3} \frac{\sin x - 2 \cos x}{2 \sin x + \cos x} dx$$

$$29. \int_0^{\pi/4} \frac{\cos x}{\sin x + 2 \cos x} dx$$

$$30. \int_0^{\pi/2} \frac{1}{\sin x - 2} dx$$

$$31. \int_{\pi/4}^{\pi/2} \frac{x \cos x}{\sin^3 x} dx$$

$$32. \int_{\pi/4}^{\pi/2} x \sin x \cos^2 x dx$$

$$33. \int_{\pi/4}^{\pi/2} \frac{x}{\sin^2 x} dx$$

$$34. \int_0^1 \frac{\sqrt{x}}{4 + 2\sqrt{x}} dx$$

$$35. \int_{-3}^0 \frac{1}{(2-x)\sqrt{1-x}} dx$$

$$36. \int_0^2 \frac{x-1}{\sqrt{x+2}} dx$$

$$37. \int_1^2 \frac{1}{x \sqrt{2x+1}} dx$$

$$38. \int_0^2 \sin(\sqrt{x}) \cos(\sqrt{x}) dx$$

$$39. \int_0^1 e^{2x} \sqrt{e^x + 1} dx$$

$$40. \int_1^2 \frac{\sqrt{x^2-1}}{x} dx$$

$$41. \int_0^{1/3} \sqrt{4-9x^2} dx$$

$$42. \int_0^2 \frac{x}{\sqrt{x^2+4}+2} dx$$

$$43. \int_{1/2}^{3/2} \frac{x-2}{\sqrt{-x^2+2x}} dx$$

$$44. \int_{\sqrt{2}}^2 \arctan \sqrt{x^2-1} dx$$

Soluzioni

A)

$$1. \left[\frac{1}{3} e^{3x+1} \right]_0^{1/3} = \frac{e^2 - e}{3}$$

$$2. \left[x \arctan(3x) - \frac{1}{6} \log(1 + 9x^2) \right]_0^{1/3} \\ = \frac{\pi}{12} - \frac{1}{6} \log 2$$

$$3. \left[\frac{1}{\pi} \sin(\pi \log x) \right]_1^e = 0$$

$$4. \left[-\frac{1}{3} \cos(x^3) \right]_0^2 = \frac{-\cos 8 + 1}{3}$$

$$5. [\arctan(\sin x)]_0^{\pi/2} = \frac{\pi}{4}$$

$$6. \left[\frac{1}{2} \exp(e^{2x} + 1) \right]_{-\log 2}^0 = \frac{e^2 - e^{5/4}}{2}$$

$$7. [(x+2) \sin x + \cos x]_{\pi/4}^{\pi/2} \\ = \frac{\pi}{2} + 2 - \frac{\pi}{4\sqrt{2}} - \frac{3}{\sqrt{2}}$$

$$8. \left[\frac{1}{4} (2x^2 - 6x + 3)e^{2x} \right]_1^2 = \frac{e^2 - e^4}{4}$$

$$9. \left[-\frac{1}{5} (25x^2 - 10x + 2)e^{-5x} \right]_0^{2/5} \\ = \frac{2 - 2e^{-2}}{5}$$

$$10. \left[(x^2 - 25) \log(x+5) - \frac{1}{2} x^2 + 5x \right]_{-2}^0 \\ = -25 \log 5 + 21 \log 3 + 12$$

$$11. \left[\frac{1}{3} x^3 \arctan x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2) \right]_0^2 \\ = \frac{8}{3} \arctan 2 - \frac{2}{3} + \frac{1}{6} \ln 5$$

$$12. \left[\frac{1}{3} x^3 \arctan(x^3) - \frac{1}{6} \ln(1+x^6) \right]_0^1 \\ = \frac{\pi}{12} - \frac{1}{6} \ln 2$$

$$13. \left[\frac{1}{3} (x^3 - 1)e^{x^3} \right]_0^2 = \frac{7e^8 + 1}{3}$$

$$14. \left[\frac{1}{2} x^2 - x + 2 \log |x+1| \right]_1^2 \\ = \frac{1}{2} + 2 \log 3 - 2 \log 2$$

$$15. \left[\frac{1}{2} \log |x| - \frac{1}{6} \log |3x+2| \right]_1^3 \\ = \frac{1}{2} \log 3 - \frac{1}{6} \log 11 + \frac{1}{6} \log 5$$

$$16. \left[\frac{1}{2} \log |x+1| - \frac{1}{2} \log |x+3| \right]_0^2 \\ = \log 3 - \frac{1}{2} \log 5$$

$$17. \left[-\frac{3}{x+2} + \log |x+2| \right]_1^2 \\ = \frac{1}{4} + \log 4 - \log 3$$

$$18. \left[\frac{1}{3} x - \frac{2}{3\sqrt{15}} \arctan \left(\sqrt{\frac{3}{5}} x \right) \right]_0^2 \\ = \frac{2}{3} - \frac{2}{3\sqrt{15}} \arctan \left(2 \sqrt{\frac{3}{5}} \right)$$

$$19. \left[\frac{1}{8} x^2 - \frac{9}{16(2x+1)} - \frac{1}{16} \log |2x+1| \right]_0^2 \\ = \frac{19}{20} - \frac{1}{16} \log 5$$

$$20. \left[\frac{1}{2} \log |x+1| + \frac{1}{2} \log |x-1| - \log |x| \right]_2^3 \\ = \frac{5}{2} \log 2 - \frac{3}{2} \log 3$$

$$21. \left[\log(x^2 - x + 2) + \frac{2}{\sqrt{7}} \arctan \frac{2x-1}{\sqrt{7}} \right]_0^3 \\ = \log 4 + \frac{2}{\sqrt{7}} \arctan \frac{5}{\sqrt{7}} + \frac{2}{\sqrt{7}} \arctan \frac{1}{\sqrt{7}}$$

$$\begin{aligned} 22. & \left[\frac{1}{6} \log(2e^x + 3) + \frac{1}{3} x \right]_0^1 \\ &= \frac{1}{6} \log(2e + 3) + \frac{1}{3} - \frac{1}{6} \log 5 \end{aligned}$$

$$\begin{aligned} 23. & \left[\frac{1}{3} e^x + \frac{5}{9} \log(3e^x + 1) \right]_0^1 \\ &= \frac{1}{3} e + \frac{5}{9} \log(3e + 1) - \frac{1}{3} - \frac{5}{9} \log 4 \end{aligned}$$

$$\begin{aligned} 24. & \left[\frac{1}{2} e^{-x} - \frac{1}{4} x + \frac{1}{4} \log |e^x - 2| \right]_0^{1/2} \\ &= \frac{1}{2} e^{-1/2} - \frac{5}{8} + \frac{1}{4} \log(2 - e^{1/2}) \end{aligned}$$

$$25. [\arctan(e^x)]_0^1 = \arctan e - \frac{\pi}{4}$$

$$26. \left[\frac{3}{4} x + \frac{1}{16} \sin(4x) \right]_{\pi/4}^{\pi/2} = \frac{3}{16} \pi$$

$$\begin{aligned} 27. & \left[\frac{1}{4} \log |\sin x - 2| - \frac{1}{4} \log |\sin x + 2| \right]_{\pi/6}^{\pi/2} \\ &= -\frac{1}{2} \log 3 + \frac{1}{4} \log 5 \end{aligned}$$

$$\begin{aligned} 28. & [-\log(2 \sin x + \cos x)]_0^{\pi/3} \\ &= -\log \frac{2\sqrt{3} + 1}{2} \end{aligned}$$

$$\begin{aligned} 29. & \left[\frac{1}{5} \log(\tan x + 2) - \frac{1}{10} \log(\tan^2 x + 1) + \right. \\ & \left. + \frac{2}{5} x \right]_0^{\pi/4} = \frac{2 \log 3 - 3 \log 2 + \pi}{10} \end{aligned}$$

$$\begin{aligned} 30. & \left[-\frac{2}{\sqrt{3}} \arctan \left(\frac{2}{\sqrt{3}} \tan \frac{x}{2} - \frac{1}{\sqrt{3}} \right) \right]_0^{\pi/2} \\ &= -\frac{2}{3\sqrt{3}} \pi \end{aligned}$$

$$31. \left[-\frac{x}{2 \sin^2 x} - \frac{1}{2} \cot x \right]_{\pi/4}^{\pi/2} = \frac{1}{2}$$

$$\begin{aligned} 32. & \left[-\frac{1}{3} x \cos^3 x + \frac{1}{3} \sin x - \frac{1}{9} \sin^3 x \right]_{\pi/4}^{\pi/2} \\ &= \frac{2}{9} + \frac{\pi}{24\sqrt{2}} - \frac{5}{18\sqrt{2}} \end{aligned}$$

$$33. [-x \cot x + \log |\sin x|]_{\pi/4}^{\pi/2} = \frac{\pi}{4} + \frac{1}{2} \log 2$$

$$\begin{aligned} 34. & \left[\frac{1}{2} x - 2\sqrt{x} + 4 \log(2 + \sqrt{x}) \right]_0^1 \\ &= -\frac{3}{2} + 4 \log 3 - 4 \log 2 \end{aligned}$$

$$\begin{aligned} 35. & [-2 \arctan \sqrt{1-x}]_{-3}^0 \\ &= -\frac{\pi}{2} + 2 \arctan 2 \end{aligned}$$

$$\begin{aligned} 36. & \left[\frac{2}{3} (x+2)^{3/2} - 6\sqrt{x+2} \right]_0^2 \\ &= -\frac{20}{3} + \frac{14}{3} \sqrt{2} \end{aligned}$$

$$\begin{aligned} 37. & \left[\log |\sqrt{2x+1} - 1| - \log |\sqrt{2x+1} + 1| \right]_1^2 \\ &= \log \frac{(\sqrt{5}-1)(\sqrt{3}+1)}{(\sqrt{5}+1)(\sqrt{3}-1)} \end{aligned}$$

$$\begin{aligned} 38. & \left[-\frac{1}{2} \sqrt{x} \cos(2\sqrt{x}) + \frac{1}{4} \sin(2\sqrt{x}) \right]_0^2 \\ &= -\frac{1}{\sqrt{2}} \cos(2\sqrt{2}) + \frac{1}{4} \sin(2\sqrt{2}) \end{aligned}$$

$$\begin{aligned} 39. & \left[\frac{2}{5} (e^x + 1)^{5/2} - \frac{2}{3} (e^x + 1)^{3/2} \right]_0^1 \\ &= \frac{2}{5} (e+1)^{5/2} - \frac{2}{3} (e+1)^{3/2} - \frac{4\sqrt{2}}{15} \end{aligned}$$

$$40. [\sqrt{x^2-1} - \arctan \sqrt{x^2-1}]_1^2 = \sqrt{3} - \frac{\pi}{3}$$

$$\begin{aligned} 41. & \left[x \sqrt{1 - \frac{9}{4} x^2} + \frac{2}{3} \arcsin \left(\frac{3}{2} x \right) \right]_0^{1/3} \\ &= \frac{1}{2\sqrt{3}} + \frac{\pi}{9} \end{aligned}$$

$$\begin{aligned} 42. & \left[\sqrt{x^2+4} - 2 \log \left(\frac{1}{2} \sqrt{x^2+4} + 1 \right) \right]_0^2 \\ &= 2\sqrt{2} - 2 \log(\sqrt{2} + 1) - 2 + 2 \log 2 \end{aligned}$$

$$\begin{aligned} 43. & \left[-\arcsin(x-1) - \sqrt{-x^2+2x} \right]_{1/2}^{3/2} \\ &= -\frac{\pi}{3} \end{aligned}$$

$$\begin{aligned} 44. & \left[x \arctan \sqrt{x^2-1} - \operatorname{arccosh} x \right]_{\sqrt{2}}^2 \\ &= \frac{2}{3} \pi - \log(2 + \sqrt{3}) - \frac{\pi}{2\sqrt{2}} + \log(\sqrt{2} + 1) \end{aligned}$$