31-10-2002

ESERCITAZIONE
(LIMITI DI SUCCESSIONI)

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Esercizio 1 Sia $\{\frac{n+2}{n+3}\}_{n\in\mathcal{N}}$. Calcolare

$$\lim_{n\to\infty}\frac{n+2}{n+3}$$

Poiché

$$\frac{n+2}{n+3}=\frac{1+o(1)}{1+o(1)}, \text{ per } n\to\infty,$$

possiamo concludere che $\lim_{n\to\infty}\frac{n+2}{n+3}=1$.

Esercizio 2 Sia $\{\frac{n-2}{n^2+2}\}_{n\in\mathcal{N}}$ Calcolare

$$\lim_{n\to\infty}\frac{n-2}{n^2+2}.$$

Poiché

$$\frac{n-2}{n^2+2}=\frac{1}{n}\cdot\frac{1+o(1)}{1+o(1)}, \text{ per } n\to\infty,$$

possiamo concludere che $\lim_{n\to\infty}\frac{n-2}{n^2+2}=0$.

Esercizio 3 Sia $\{\frac{n^2-2n+4}{3-n}\}_{n\in\mathcal{N}}$ Calcolare

$$\lim_{n\to\infty}\frac{n^2-2n+4}{3-n}.$$

Poiché

$$\frac{n^2-2n+4}{3-n}=n\frac{1+o(1)}{-1+o(1)}, \text{ per } n\to\infty,$$

possiamo concludere che $\lim_{n\to\infty}\frac{n^2-2n+4}{3-n}=-\infty$.

Esercizio 4 Sia $\{\frac{4n^2+1}{-2n^2+1}\}_{n\in\mathcal{N}}$. Calcolare

$$\lim_{n\to\infty}\frac{4n^2+1}{-2n^2+1}.$$

Poiché

$$\frac{4n^2+1}{-2n^2+1}=\frac{4+o(1)}{-2+o(1)}, \text{ per } n\to\infty,$$

possiamo concludere che $\lim_{n\to\infty}\frac{4n^2+1}{-2n^2+1}=-2$.

Esercizio 5 Sia $\{\frac{n^3+3n}{n^3+4n^5}\}_{n\in\mathcal{N}}$. Calcolare

$$\lim_{n\to\infty}\frac{n^3+3n}{n^3+4n^5}, \ [R=0].$$

Esercizio 6 Sia $\{\frac{\sum_{k=0}^m a_k n^k}{\sum_{j=0}^p b_j n^j}\}_{n\in\mathcal{N}}$. Calcolare, al variare di $m,p\in\mathcal{N}\cup\{0\}$, e nell'ipotesi in cui $a_m\neq 0$, $b_p\neq 0$,

$$\lim_{n\to\infty}\frac{\sum_{k=0}^m a_k n^k}{\sum_{j=0}^p b_j n^j}.$$

Se $m > p$, allora

$$\frac{\sum_{k=0}^{m} a_k n^k}{\sum_{j=0}^{p} b_j n^j} = n^{m-p} \frac{a_m + o(1)}{b_p + o(1)}, \text{ per } n \to \infty,$$

pertanto

$$\lim_{n\to\infty} \frac{\sum_{k=0}^{m} a_k n^k}{\sum_{j=0}^{p} b_j n^j} = \text{sgn}(\frac{a_m}{b_p}) \cdot \infty.$$

Se $m = p$, allora

$$\frac{\sum_{k=0}^{m} a_k n^k}{\sum_{j=0}^{m} b_j n^j} = \frac{a_m + o(1)}{b_m + o(1)}, \text{ per } n \to \infty,$$

pertanto

$$\lim_{n\to\infty} \frac{\sum_{k=0}^{m} a_k n^k}{\sum_{j=0}^{m} b_j n^j} = \frac{a_m}{b_p}.$$

Infine se $m < p$

$$\frac{\sum_{k=0}^{m} a_k n^k}{\sum_{j=0}^{p} b_j n^j} = n^{m-p} \frac{a_m + o(1)}{b_p + o(1)}, \text{ per } n \to \infty,$$

pertanto

$$\lim_{n\to\infty} \frac{\sum_{k=0}^{m} a_k n^k}{\sum_{j=0}^{p} b_j n^j} = 0.$$

Esercizio 7 Sia $\{\sqrt{2n+1} - \sqrt{2n-6}\}_{n\in\mathcal{N}}$. Calcolare

$$\lim_{n\to\infty} \Big(\sqrt{2n+1} - \sqrt{2n-6}\Big).$$

Poiché

$$\sqrt{2n+1} - \sqrt{2n-6} = (\sqrt{2n+1} - \sqrt{2n-6}) \frac{\sqrt{2n+1} + \sqrt{2n-6}}{\sqrt{2n+1} + \sqrt{2n-6}} = \frac{2n+1-2n+6}{\sqrt{2n+1} + \sqrt{2n-6}}$$

$$= \frac{7}{n^{1/2}(\sqrt{2+o(1)} + \sqrt{2+o(1)})}, \text{ per } n \to \infty,$$

possiamo concludere che

$$\lim_{n\to\infty} \Big(\sqrt{2n+1} - \sqrt{2n-6}\Big) = 0.$$

Esercizio 8 Sia $\{\sqrt{4n^4+3} - \sqrt{4n^4+88}\}_{n\in\mathcal{N}}$. Calcolare

$$\lim_{n\to\infty} \Big((4n^4+3)^{1/3} - (4n^4+88)^{1/3}\Big).$$

Poiché

$$\begin{aligned}
& (4n^4+3)^{1/3} - (4n^4+88)^{1/3} \\
= & \Big((4n^4+3)^{1/3} - (4n^4+88)^{1/3}\Big) \frac{(4n^4+3)^{2/3} + (4n^4+3)^{1/3}(4n^4+88)^{1/3} + (4n^4+88)^{1/3}}{(4n^4+3)^{2/3} + (4n^4+3)^{1/3}(4n^4+88)^{1/3} + (4n^4+88)^{2/3}} \\
= & \frac{4n^4+3-4n^4-88}{(4n^4+3)^{2/3} + (4n^4+3)^{1/3}(4n^4+88)^{1/3} + (4n^4+88)^{2/3}},
\end{aligned}$$

si conclude che

$$\lim_{n\to\infty} \Big((4n^4+3)^{1/3} - (4n^4+88)^{1/3}\Big) = 0.$$

Esercizio 9 Sia $\{\frac{n^{13/6}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\sqrt{2n+1}-\sqrt{2n-6}}\}_{n\in\mathcal{N}}$. Calcolare

$$\lim_{n\to\infty}\frac{n^{13/6}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\sqrt{2n+1}-\sqrt{2n-6}}.$$

Poiché

$$\begin{aligned}
&\frac{n^{13/6}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\sqrt{2n+1}-\sqrt{2n-6}}\\
=\;&\frac{n^{13/6}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\sqrt{2n+1}-\sqrt{2n-6}}\frac{(4n^4+3)^{2/3}+(4n^4+3)^{1/3}(4n^4+88)^{1/3}+(4n^4+88)^{2/3}}{(4n^4+3)^{2/3}+(4n^4+3)^{1/3}(4n^4+88)^{1/3}+(4n^4+88)^{2/3}}\\
=\;&\frac{n^{13/6}(4n^4+3-4n^4-88)}{(\sqrt{2n+1}-\sqrt{2n-6})((4n^4+3)^{2/3}+(4n^4+3)^{1/3}(4n^4+88)^{1/3}+(4n^4+88)^{2/3})}\\
=\;&\frac{-85n^{13/6}}{(\sqrt{2n+1}-\sqrt{2n-6})((4n^4+3)^{2/3}+(4n^4+3)^{1/3}(4n^4+88)^{1/3}+(4n^4+88)^{2/3})}\\
=\;&\frac{-85n^{13/6}}{(\sqrt{2n+1}-\sqrt{2n-6})((4n^4+3)^{2/3}+(4n^4+3)^{1/3}(4n^4+88)^{1/3}+(4n^4+88)^{2/3})}\\
\times\;&\frac{\sqrt{2n+1}+\sqrt{2n-6}}{\sqrt{2n+1}+\sqrt{2n-6}}\\
=\;&\frac{-85n^{13/6}}{2n+1-2n+6}\frac{(\sqrt{2n+1}+\sqrt{2n-6})}{(4n^4+3)^{2/3}+(4n^4+3)^{1/3}(4n^4+88)^{1/3}+(4n^4+88)^{2/3})}\\
=\;&\frac{-85n^{13/6}}{7}\frac{\sqrt{n}(\sqrt{2+\frac{1}{n}}+\sqrt{2-\frac{6}{n}})}{n^{8/3}\big[(4+\frac{3}{n^4})^{2/3}+(4+\frac{3}{n^4})^{1/3}(4+\frac{88}{n^4})^{1/3}+(4+\frac{88}{n^4})^{2/3})\big]}\\
=\;&\frac{-85n^{\frac{13}{6}+\frac{1}{2}-\frac{8}{3}}}{7}\frac{\sqrt{2+\frac{1}{n}}+\sqrt{2-\frac{6}{n}}}{(4+\frac{3}{n^4})^{2/3}+(4+\frac{3}{n^4})^{1/3}(4+\frac{88}{n^4})^{1/3}+(4+\frac{88}{n^4})^{2/3})}.
\end{aligned}$$

Quindi

$$\lim_{n\to\infty}\frac{n^{13/6}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\sqrt{2n+1}-\sqrt{2n-6}}=\frac{-170\sqrt{2}}{21\cdot(4)^{2/3}}.$$

Esercizio 10 Sia $\{\frac{n^{17/3}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\cos(4n)+\sqrt{2n^2+1}-\sqrt{2n-6}}\}_{n\in\mathcal{N}}$. Calcolare

$$\lim_{n\to\infty}\frac{n^{11/3}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\cos(4n)+\sqrt{2n^2+1}-\sqrt{2n-6}}.$$

Poiché

$$\begin{aligned}
&\frac{n^{11/3}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\cos(4n)+\sqrt{2n^2+1}-\sqrt{2n-6}}=\frac{n^{11/3}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\cos(4n)+\frac{2n^2+1-2n+6}{\sqrt{2n^2+1}+\sqrt{2n-6}}}\\
=\;&\frac{n^{11/3}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\cos(4n)+n\frac{2-2\frac{1}{n}+\frac{7}{n^2}}{\sqrt{2+\frac{1}{n^2}}+\sqrt{\frac{2}{n}-\frac{6}{n^2}}}}=\frac{n^{8/3}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\frac{\cos(4n)}{n}+\frac{2-2\frac{1}{n}+\frac{7}{n^2}}{\sqrt{2+\frac{1}{n^2}}+\sqrt{\frac{2}{n}-\frac{6}{n^2}}}}\\
=\;&-85\frac{n^{8/3}}{\frac{\cos(4n)}{n}+\frac{2-2\frac{1}{n}+\frac{7}{n^2}}{\sqrt{2+\frac{1}{n^2}}+\sqrt{\frac{2}{n}-\frac{6}{n^2}}}}\frac{1}{\big[(4n^4+3)^{2/3}+(4n^4+3)^{1/3}(4n^4+88)^{1/3})+(4n^4+88)^{2/3}\big]}.
\end{aligned}$$

Pertanto

$$\lim_{n\to\infty} \frac{n^{11/3}((4n^4+3)^{1/3}-(4n^4+88)^{1/3})}{\cos(4n)+\sqrt{2n^2+1}-\sqrt{2n-6}} = \frac{-85}{3\cdot 2^{11/6}}.$$