SOLUZIONI DEL COMPITO del 10/12/2002 Corsi: Meccanica, Elettrica e Telecomunicazioni del prof. Obrecht (Commissione: Obrecht, Ferrari)

1.
$$\lim_{x\to 0} \frac{\sqrt{1+4x^2}-\cos(2x)-4x^2}{x\cos^3 x\,(3x-\operatorname{sen}(3x))} = -\frac{16}{27}.$$

2.
$$f(x) = \frac{|x-4|}{x^2-3x+2}$$

$\operatorname{dom}(f) = \mathbf{R} \setminus \{1,2\}$,

$$\forall x \in \mathbf{R} \setminus \{1,2,4\}, \quad f'(x) = -\operatorname{sgn}(x-4)\frac{x^2-8x+10}{(x-1)^2(x-2)^2}$$

. Si ha:
$$1 < 4-\sqrt{6} < 2 < 4 < 4+\sqrt{6}$$

Pertanto f è:

- strettamente crescente in $(-\infty,1)$, in $\left(1, 4-\sqrt{6}\right]$, in $\left[4, 4+\sqrt{6}\right]$
- strettamente decrescente in $\left[4-\sqrt{6}, 2\right)$, in $(2,4]$, $\left[4+\sqrt{6}, +\infty\right)$

3.
$$h(x) = \log\left|\frac{x^2-4}{3x+1}\right|$$
$$\forall x \in \mathbf{R} \setminus \left\{-2, -\frac{1}{3}, 2\right\}, \quad h'(x) = \frac{3x^2+2x+12}{(x^2-4)(3x+1)}$$

Pertanto

- h è strettamente crescente in $\left(-2, -\frac{1}{3}\right)$, in $(2,+\infty)$
- h è strettamente decrescente in $(-\infty,-2)$, in $\left(-\frac{1}{3}, 2\right)$.

4.
$$\int_0^{\frac{\pi}{4}} (x+5)\operatorname{sen}(2x)\, dx = \frac{11}{4}.$$

5.
$$\int_0^1 \frac{7x+7}{\sqrt{x^2+2x+3}}\,dx = 7\left(\sqrt{6}-\sqrt{3}\right).$$

6.
$$\left(D\,(x+5)\,\sqrt{x^2+x+2}\right)_{x=c} = \sqrt{c^2+c+2} + \frac{(2x+1)\,(c+5)}{2\sqrt{c^2+c+2}} =$$
$$= \frac{4c^2+13c+9}{2\sqrt{c^2+c+2}}.$$

7.
$$\left(D\,(x+5)^{\text{ sen }\left(2x^2\right)}\right)_{x=\sqrt{\frac{\pi}{4}}} = 1.$$

Infatti,
$$\left(D\,(x+5)^{\text{ sen }\left(2x^2\right)}\right)(x) = (x+5)^{\text{ sen }\left(2x^2\right)}\left(4x\cos\left(2x^2\right)\log\,(x+5) + \frac{\text{sen }\left(2x^2\right)}{x+5}\right)$$

8.
$$\lim_{x\to 5} \frac{x^2-3x-10}{x^3-14x^2+44x+50} = -\frac{7}{30}.$$

Infatti,
$$\frac{x^2-3x-10}{x^3-14x^2+44x+50} = \frac{(x-5)\,(x+2)}{(x-5)\,(x+1)\,(x-10)}$$

9. Posto
$$g(x) = (x+5)\exp\left(\frac{x}{x-2}\right)$$
sapendo che
$$g'(x) = \exp\left(\frac{x}{x-2}\right)\frac{x^2-6x-6}{(x-2)^2},$$
risulta che g è convessa in
$$\left(\frac{9}{8},2\right) \quad \text{e in} \quad (2,+\infty).$$

10.
$$\lim_{x\to+\infty} \frac{5+x^3}{\log^3(2x)}\left(\text{ sen }\left(\frac{\log x}{x}\right) - \frac{\log x}{x}\right) = -\frac{1}{6}.$$