

Date rette proiettive $\mathcal{P}, \mathcal{P}'$
dati in \mathcal{P} (rispetto a un $\overline{\mathcal{R}}$)

$$A_0 \equiv (1, 1), A_1 \equiv (1, 3), U \equiv (5, 6)$$

in \mathcal{P}' (rispetto a un $\overline{\mathcal{R}'}$)

$$A'_0 \equiv (1, 2), A'_1 \equiv (3, 5), U' \equiv (20, 30)$$

a) Si verifichi che

$$\mathcal{R} = (A_0, A_1, U) \text{ ed}$$

$$\mathcal{R}' = (A'_0, A'_1, U') \text{ sono}$$

riferimenti proiettivi
delle rispettive rette.

b) Si scriva l'eq. matr.
della proiettività $w: \mathbb{P}^1 \rightarrow \mathbb{P}^1$

t. c. $w(A_0) = A'_0, w(A_1) = A'_1, w(U) = U'$

c) Dato $Q = (4, -3)$ si trovi

$w(Q)$.

a) $R = (1, 2, 3)$ fatto
 $R' = (2, 5, 30)$

$$\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} \neq 0 \quad \begin{vmatrix} 1 & 20 \\ 2 & 30 \end{vmatrix} \neq 0 \quad \begin{vmatrix} 3 & 20 \\ 5 & 30 \end{vmatrix} \neq 0$$

\mathcal{Q}' è un rif. pro.

Base \mathcal{B} normalizzata risp.
a \mathcal{Q} , $\mathcal{B} = \left((9, 9), (1, 3) \right)$

Cerca una base \mathcal{B}' normalizzata risp. ad \mathcal{Q}' ,
 $\alpha(1, 2) + \beta(3, 5) = (20, 30)$

$$\begin{cases} \alpha + 3\beta = 20 \\ 2\alpha + 5\beta = 30 \end{cases}$$

$$\begin{cases} \alpha + 3\beta = 20 \\ -\beta = -10 \end{cases} \quad \begin{cases} \alpha = -10 \\ \beta = 10 \end{cases}$$

$$B' = \left(\cancel{(-10, -20)}, \cancel{(30, 50)} \right) \\ \left((-1, -2), (3, 5) \right)$$

$$M \cdot \begin{pmatrix} 9 & 1 \\ 9 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ -2 & 5 \end{pmatrix}$$

$X \qquad Y$

$$M = Y \cdot X^{-1}$$

$$|X| = 18 \quad X^{-1} = \frac{1}{18} \begin{pmatrix} 3 & -1 \\ -9 & 9 \end{pmatrix}$$

$$M = \begin{pmatrix} -1 & 3 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -9 & 9 \end{pmatrix} \frac{1}{18} =$$
$$= \frac{1}{18} \begin{pmatrix} -30 & 28 \\ -51 & 47 \end{pmatrix}$$

$$\lambda \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} -30 & 28 \\ -51 & 47 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$

$$c) \omega(Q) \equiv \begin{pmatrix} -30 & 28 \\ -51 & 47 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \end{pmatrix} =$$
$$= \begin{pmatrix} -204 \\ -345 \end{pmatrix}$$

$$\lambda \begin{pmatrix} y_0 \\ y_1 \end{pmatrix} = \begin{pmatrix} -30 & 28 \\ -51 & 47 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$

$$\begin{pmatrix} -30 & 28 \\ -51 & 47 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} =$$

$$= -2 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} -30 & 28 \\ -51 & 47 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 54 \\ 90 \end{pmatrix} = 18 \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

