Iperq. Q non spec. di discriminante A.

Cerco il generico iperpiano diametrale: è l'iperpiano polare del generico punto improprio $(0, l_1, \dots, l_n)$

$$(X_0 \cdots X_n)\begin{pmatrix} a_0^0 & a_1^0 & \cdots & a_n^0 \\ a_0^1 & a_1^1 & \cdots & a_n^1 \\ \cdots & \cdots & \cdots & \cdots \\ a_0^n & a_1^n & \cdots & a_n^n \end{pmatrix} \cdot \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$$

$$b_0 X_0 + b_1 X_1 + \cdots + b_n X_n = 0$$

$$\begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} a_0^0 & \cdots & a_n^0 \\ a_0^1 & a_1^1 \cdots & a_n^1 \\ \cdots & \cdots & \cdots \\ a_0^n & a_1^n \cdots & a_n^n \end{pmatrix} \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = M_{00} \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

ortogonalità:

$\exists \lambda \neq 0$ t.c. $\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \lambda \cdot \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$

cioè $\exists \lambda \neq 0$ t.c. $M_{00} \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$

1/7/2009 Es. 2b

$Q: x^2+2y^2-z^2-2x+2z=0$

Trovarne un piano principale

$$A = \begin{pmatrix} 0 & -1 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} (t-1) & 0 & 0 \\ 0 & (t-2) & 0 \\ 0 & 0 & (t+1) \end{pmatrix}$$

M_{00}

Autovalori: $1, -1, 2$

Scelgo autoval. 2

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{cases} l=0 \\ 0=0 \\ n=0 \end{cases}$$

autospazio $U_2 = \{(0,\alpha,0) \mid \alpha \in \mathbb{R}\}$

una base: $\{(0,1,0)\}$

Polo di un piano princ.

$(0,0,1,0)$

$$(0\ 0\ 1\ 0) \cdot \begin{pmatrix} 0 & -1 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$$\boxed{2y=0}$$

Trovare i piani principali di Q:

$$7x^2 - 48xy - 7y^2 + 25z^2 - 10z + 2 = 0$$

$$A = \begin{pmatrix} 2 & 0 & 0 & -5 \\ 0 & 7 & -24 & 0 \\ 0 & -24 & -7 & 0 \\ -5 & 0 & 0 & 25 \end{pmatrix}$$

$$M_{00} = \begin{pmatrix} 7 & -24 & 0 \\ -24 & -7 & 0 \\ 0 & 0 & 25 \end{pmatrix} \qquad \begin{vmatrix} (t-7) & 24 & 0 \\ 24 & (t+7) & 0 \\ 0 & 0 & (t-25) \end{vmatrix} =$$

$$= (t-25)\left((t-7)(t+7) - 24^2\right) =$$

$$= (t-25)(t^2 - 49 - 576) = (t-25)(t^2 - 625) =$$

$$= (t-25)^2 (t+25)$$

$$t = -25 \qquad \begin{pmatrix} \cancel{-32} & \cancel{24} & \cancel{0} \\ 24 & -18 & 0 \\ 0 & 0 & -50 \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} \cancel{0} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 4l - 3m = 0 \\ n = 0 \end{cases} \quad (l, m, n) \sim \{(3, 4, 0)\}$$

base di V_{-25}

$$\begin{pmatrix} 0 & 3 & 4 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & -5 \\ 0 & 7 & -24 & 0 \\ 0 & -24 & -7 & 0 \\ -5 & 0 & 0 & 25 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$$\boxed{-75x - 100y = 0}$$

$$t = 25 \quad \begin{pmatrix} 18 & 24 & 0 \\ \cancel{24} & \cancel{32} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 0 \\ \cancel{0} \\ \cancel{0} \end{pmatrix}$$

$$18l + 24m = 0$$

$$3l + 4m = 0$$

$$U_{25} = \{(4\alpha, -3\alpha, \beta) \mid \alpha, \beta \in \mathbb{R}\}$$

una base per U_{25}:

$$\{(4, -3, 0), (0, 0, 1)\}$$

π_1 polare di $(0, 4, -3, 0)$

π_2 " " $(0, 0, 0, 1)$

$$\begin{pmatrix} 0 & 4 & -3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 0 & 0 & -5 \\ 0 & 7 & -24 & 0 \\ 0 & -24 & -7 & 0 \\ -5 & 0 & 0 & 25 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$$\begin{cases} \Pi_1 : & 100x - 75y = 0 \\ \Pi_2 : & -5 + 25z = 0 \end{cases}$$