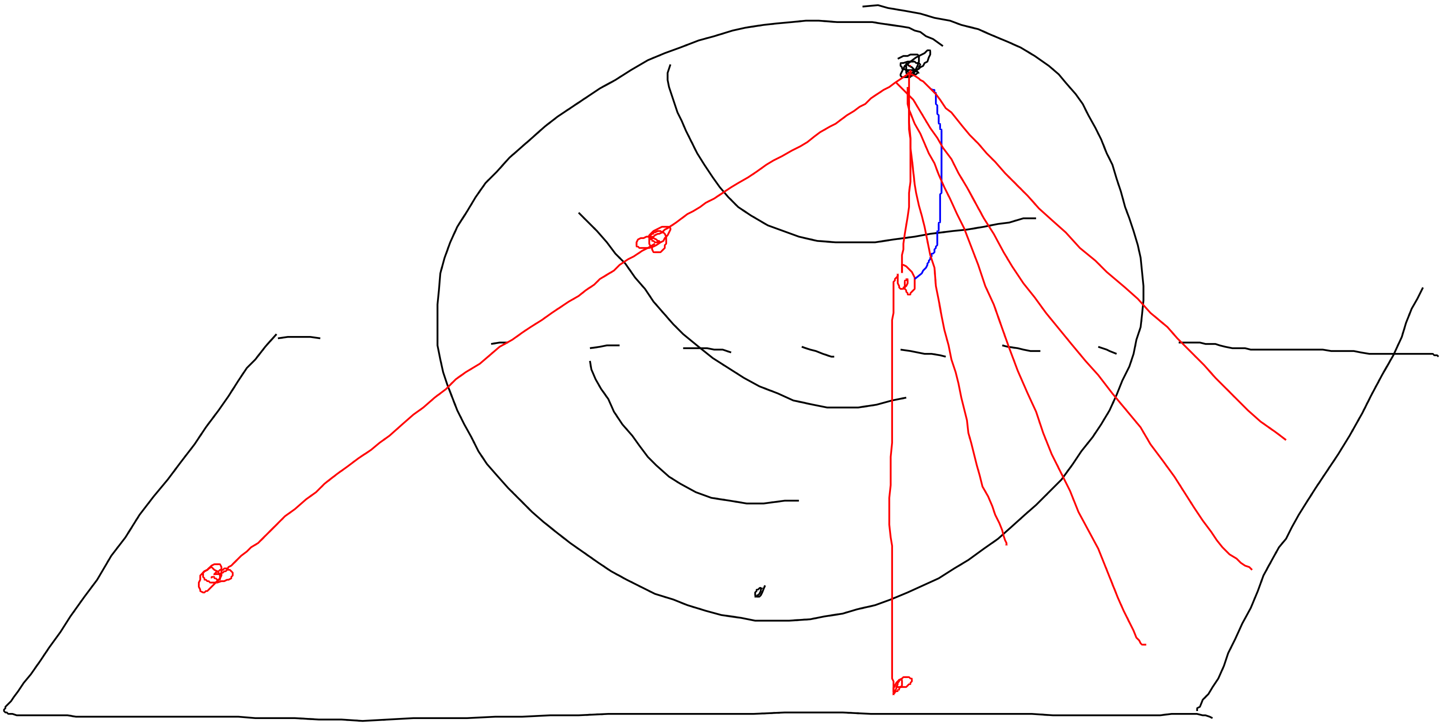

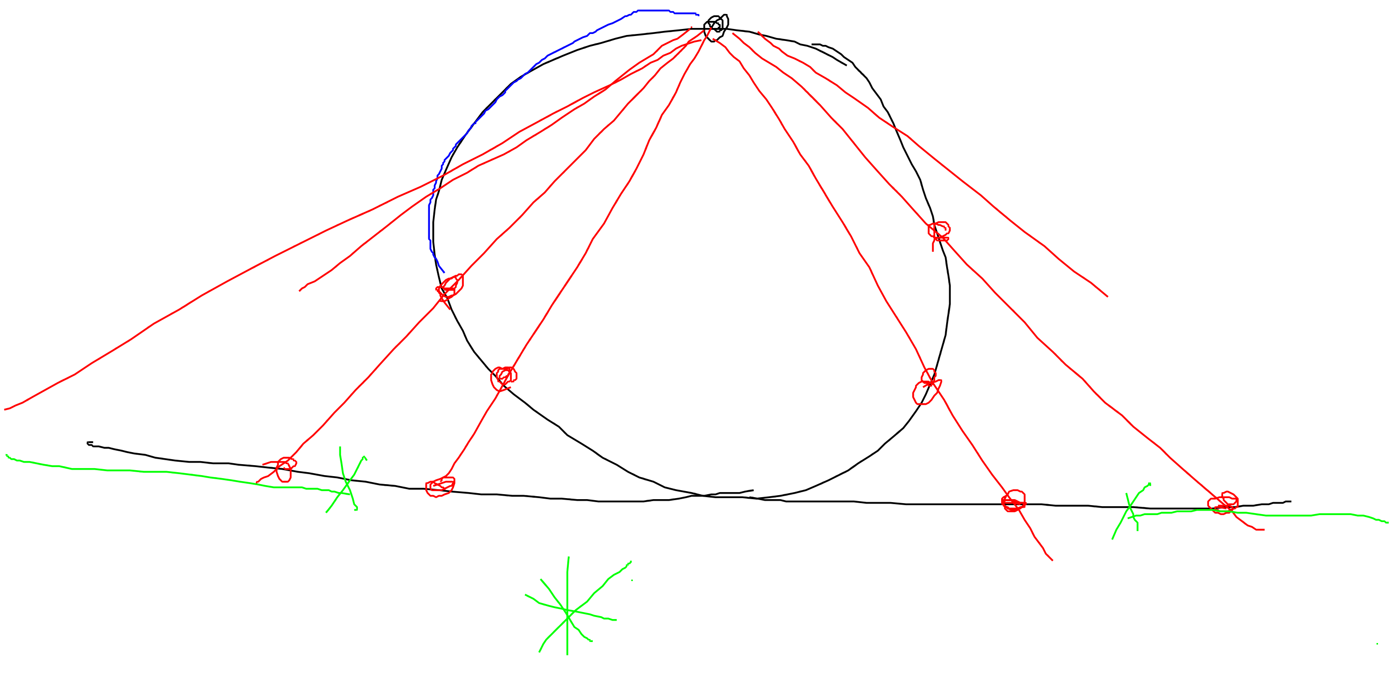

$W = \mathbb{R}^3$

Com'è fatta una forma lineare su $\mathbb{R}^3$?

$$f_{(6,-5,8)} : \mathbb{R}^3 \longrightarrow \mathbb{R}$$
$$(x,y,z) \longmapsto 6x - 5y + 8z$$

$$f_{(a,b,c)} : \mathbb{R}^3 \longrightarrow \mathbb{R}$$
$$(x,y,z) \longmapsto ax + by + cz$$

$$3f_{(6,-5,8)} : \mathbb{R}^3 \longrightarrow \mathbb{R}$$
$$(x,y,z) \longmapsto 3f_{(6,-5,8)}(x,y,z) =$$
$$= 3\cdot(6x - 5y + 8z) =$$
$$= 18x - 15y + 24z$$

$$3f_{(6,-5,8)} = f_{3\cdot(6,-5,8)}$$

$$f_{(1,2,3)} : \mathbb{R}^3 \longrightarrow \mathbb{R}$$
$$(x,y,z) \longmapsto x+2y+3z$$

$$\left(f_{(6,-5,8)} + f_{(1,2,3)}\right) : \mathbb{R}^3 \longrightarrow \mathbb{R}$$
$$(x,y,z) \longmapsto f_{(6,-5,8)}(x,y,z) + f_{(1,2,3)}(x,y,z) =$$
$$= 6x - 5y + 8z + x + 2y + 3z$$
$$= 7x - 3y + 11z$$

$$f_{(6,-5,8)} + f_{(1,2,3)} = f_{\left((6,-5,8)+(1,2,3)\right)}$$

$$f_{(6,-5,8)} \sim f_{(18,-15,24)}$$

$$6x - 5y + 8z = 0$$

$$18x - 15y + 24z = 0$$

$v_1, \dots, v_h$ $\in V$-sp.v. lin. dipendenti

cioè $\exists\, \lambda_1, \dots, \lambda_h$ non tutti nulli tali che

$$\lambda_1 v_1 + \dots + \lambda_h v_h = \bar{0}_V$$

Siano $v'_1 = \alpha_1 v_1, \dots, v'_h = \alpha_h v_h$

$\alpha_1, \dots, \alpha_h$ tutti non nulli

Tesi: $\exists\, \mu_1, \dots, \mu_h$ non tutti nulli tali che

$$\mu_1 v'_1 + \dots + \mu_h v'_h = \bar{0}_V$$

DIM - Pongo $\mu_i = \dfrac{\lambda_i}{\alpha_i}$

$$\mu_1 v'_1 + \dots + \mu_h v'_h =$$

$$= \frac{\lambda_1}{\cancel{\alpha_1}}(\cancel{\alpha_1} v_1) + \dots + \frac{\lambda_h}{\cancel{\alpha_h}}(\cancel{\alpha_h} v_h) =$$

$$= \lambda_1 v_1 + \dots + \lambda_h v_h = \bar{0}_V$$

$\dim V = n+1$ $\qquad$ V' s.sp.vett

$\dim V' = h+1$

In forma cartesiana V' è rappresentato da $(n+1)-(h+1) = n+1-h-1 = n-h$ equazioni lineari omogenee indipendenti.

$$V \xrightarrow{\Phi_{\mathcal{B}}} \mathbb{R}^n$$
base — sp.V.

$$(V, \langle , \rangle) \xrightarrow{\Phi_{\mathcal{B}}} (\mathbb{R}^n, \langle , \rangle_{nat})$$
base ortonorm — sp.V.eucl.

$$(V, V, \tau) \xrightarrow{\Phi_{\mathcal{R}}} (\mathbb{R}^n, \mathbb{R}^n, \tau)$$
rif. affine — sp.aff.

$$((V, \langle , \rangle), V, \tau) \xrightarrow{\Phi_{\mathcal{R}}} ((\mathbb{R}^n, \langle , \rangle_{nat}), \mathbb{R}^n, \tau)$$
rif. cart. — sp. eucl

$$(V, P, \vartheta) \xrightarrow{?} \left(\mathbb{R}^{n+1}, \underset{\sim}{\underline{\mathbb{R}^{n+1} - \{(0)\}}}, \vartheta\right)$$