In $\mathbb{P}$ siano

$B_0 \equiv_S (1,2)$, $B_1 \equiv_S (4,9)$, $\overline{U} \equiv_S (1,1)$

In $\mathbb{P}'$ siano

$B_0' \equiv_{S'} (1,3)$, $B_1' \equiv_{S'} (2,4)$, $\overline{U}' \equiv_{S'} (1,4)$

a) si verifichi che

$\overline{S} = (B_0, B_1, \overline{U})$ ed $\overline{S}' = (B_0', B_1', \overline{U}')$

sono rif. proiettivi di $\mathbb{P}$, $\mathbb{P}'$ rispettivamente.

b) Si scriva, rispetto ad S ed S', la rappresentazione matriciale della proiettività

$$\omega : \mathbb{P} \to \mathbb{P}'$$

tale che sia

$\omega(B_0) = B_0'$, $\omega(B_1) = B_1'$, $\omega(\overline{U}) = \overline{U}'$

Una base normalizzata risp. ad $\overline{S}$: $\mathcal{B} = (\overline{w}_0, \overline{w}_1)$

$\overline{w}_0 =_{\mathcal{B}} (5,10)$, $\overline{w}_1 =_{\mathcal{B}} (-4,-9)$

Cerco una base $\mathcal{B}'$ normalizzata rispetto ad $\overline{S}'$.

$$\alpha(1,3)+\beta(2,4) = (1,4)$$

$$\begin{cases}\alpha + 2\beta = 1\\ 3\alpha + 4\beta = 4\end{cases} \qquad \begin{cases}\alpha + 2\beta = 1\\ -2\beta = 1\end{cases}$$

$$\begin{cases}\alpha = 1 - 2\beta = 1+1 = 2\\ \beta = -\frac{1}{2}\end{cases}$$

$$\overline{w}_0' = (2,6)_{\mathcal{B}'},\ \overline{w}_1' = (-1,-2)_{\mathcal{B}'} \qquad \mathcal{B}' = (\overline{w}_0', \overline{w}_1')$$

$$M \cdot \underbrace{\begin{pmatrix}5 & -4\\ 10 & -9\end{pmatrix}}_{X} = \underbrace{\begin{pmatrix}2 & -1\\ 6 & -2\end{pmatrix}}_{Y}$$

$$|X| = -5 \qquad X^{-1} = -\frac{1}{5}\begin{pmatrix}-9 & 4\\ -10 & 5\end{pmatrix}$$

$$M = Y \cdot X^{-1} = \begin{pmatrix}2 & -1\\ 6 & -2\end{pmatrix} \cdot \left(-\frac{1}{5}\right)\begin{pmatrix}-9 & 4\\ -10 & 5\end{pmatrix} =$$

$$= \left(-\frac{1}{5}\right)\begin{pmatrix}-8 & 3\\ -34 & 14\end{pmatrix} \quad \boxed{A \cdot \begin{pmatrix}x'\\ y'\end{pmatrix} = \begin{pmatrix}-8 & 3\\ -34 & 14\end{pmatrix}}$$

Verifico:

$$\begin{pmatrix}-8 & 3\\ -34 & 14\end{pmatrix}\cdot\begin{pmatrix}1\\ 2\end{pmatrix}=\begin{pmatrix}-2\\ -6\end{pmatrix}=-2\cdot\begin{pmatrix}1\\ 3\end{pmatrix}$$

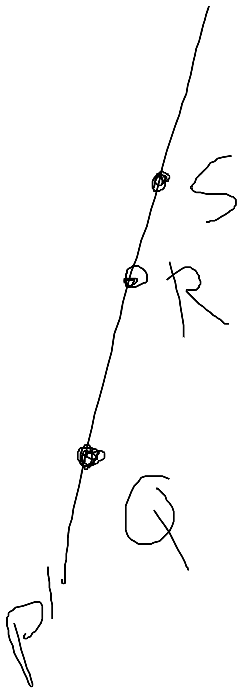
$$\cdot\begin{pmatrix}4\\ 9\end{pmatrix}=\begin{pmatrix}-5\\ -10\end{pmatrix}=-\frac{5}{2}\begin{pmatrix}2\\ 4\end{pmatrix}$$

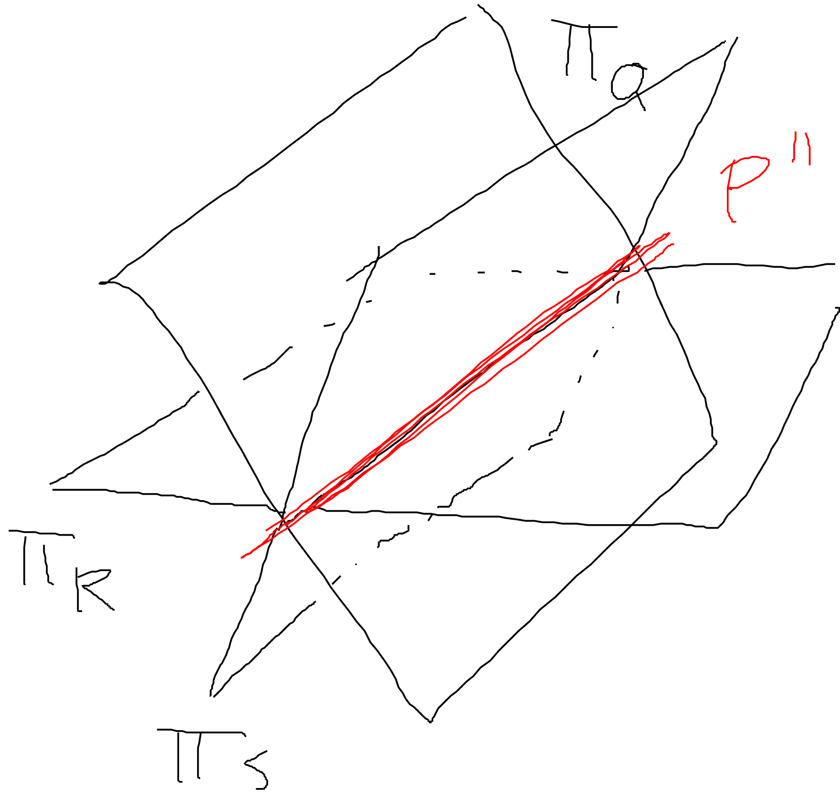
$$\begin{pmatrix}1\\ 1\end{pmatrix}=\begin{pmatrix}-5\\ -20\end{pmatrix}=-5\begin{pmatrix}1\\ 4\end{pmatrix}$$

$\mathbb{RP}^2$ $\qquad\qquad$ $\mathbb{R}^3$

$$P=(5,6,-2)\longmapsto \Pi_p: 5X_0+6X_1-2X_2=0$$

$$\overset{2}{(10,12,-4)} \qquad 10X_0+12X_1-4X_2=0$$


S
R
Q
p'


Π_Q
p''
Π_R
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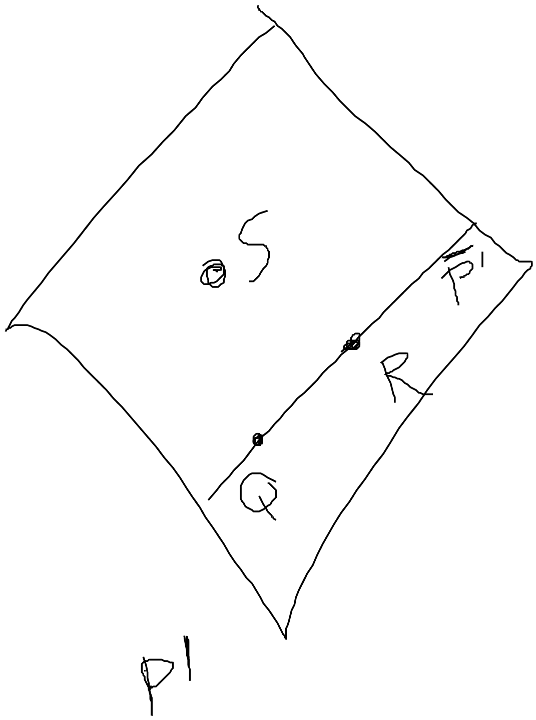
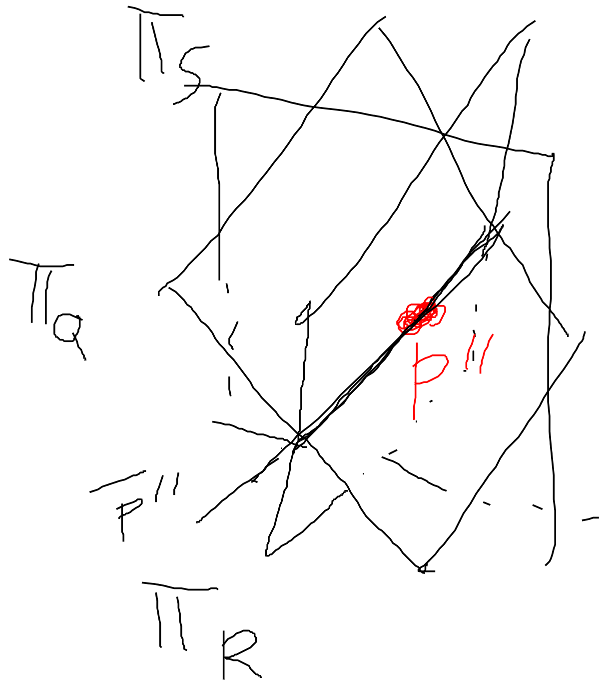

S
R
Q
P'
P'


Π_S
Π_Q
Π_R
P''
P''