Trovare un piano principale della quadrica

$$x^2+y^2+9z^2-4xy+z=0$$

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$

M_{00}

$$\begin{vmatrix} (t-1) & 2 & 0 \\ 2 & (t-1) & 0 \\ 0 & 0 & (t-9) \end{vmatrix} = (t-9)\left((t-1)^2-2^2\right) =$$

$$= (t-9)(t-3)(t+1)$$

autovalori: $-1, 3, 9$

$t = -1$

U_{-1}:

$$\begin{pmatrix} \cancel{-2} & \cancel{2} & \cancel{0} \\ 2 & -2 & 0 \\ 0 & 0 & -10 \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} \cancel{0} \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 2l - 2m = 0 \\ -10n = 0 \end{cases} \quad \begin{cases} l = m \\ n = 0 \end{cases}$$

Base: $\{(1,1,0)\}$

$(0,1,1,0)$ è polo di un piano principale

$$\pi : \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$$-x - y = 0$$

Trovare il polo del piano $x = 0$. (piano yz)

Prendo 3 punti lin. indip. sul piano

$O \equiv (1,0,0,0)$ $Y_\infty \equiv (0,0,1,0)$

$Z_\infty \equiv (0,0,0,1)$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{cases} 2x_0 = 0 \\ -2x_1 + x_2 = 0 \\ 9x_3 = 0 \end{cases} \quad \begin{cases} x_0 = 0 \\ x_2 = 2x_1 \\ x_3 = 0 \end{cases}$$

Polo: $P \equiv (0, 1, 2, 0)$

Verificare che la quadrica

$$x^2 + y^2 - 6z^2 + 4x - 2z - 1 = 0$$

è di rotazione e trovarne il sottospazio di rotazione

$$A = \begin{pmatrix} -1 & 2 & 0 & -1 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -6 \end{pmatrix}$$

$M_{\bar{a}\bar{a}}$

$$\begin{vmatrix} (t-1) & 0 & 0 \\ 0 & (t-1) & 0 \\ 0 & 0 & (t+6) \end{vmatrix}$$

Autovalori $\qquad m_a = m_g$

1	2
-6	1

$$t = 1 \quad \begin{pmatrix} \cancel{0 \ 0 \ 0} \\ \cancel{0 \ 0 \ 0} \\ 0 \ 0 \ 7 \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} \cancel{0} \\ \cancel{0} \\ 0 \end{pmatrix}$$

U_1: $n = 0$

Base: $\{(1,0,0), (0,1,0)\}$

(ogni $(\lambda, \mu, 0) \neq (0,0,0)$ è autovettore relativo a $t=1$)

$$(0\ 1\ 0\ 0)$$
$$(0\ 0\ 1\ 0)$$

$$\begin{pmatrix} -1 & 2 & 0 & -1 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & -6 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$$2 + x = 0$$

$$y = 0$$

Questi sono due piani principali relativi all'autovalore multiplo $t=1$, ma anche ogni loro combinazione lineare

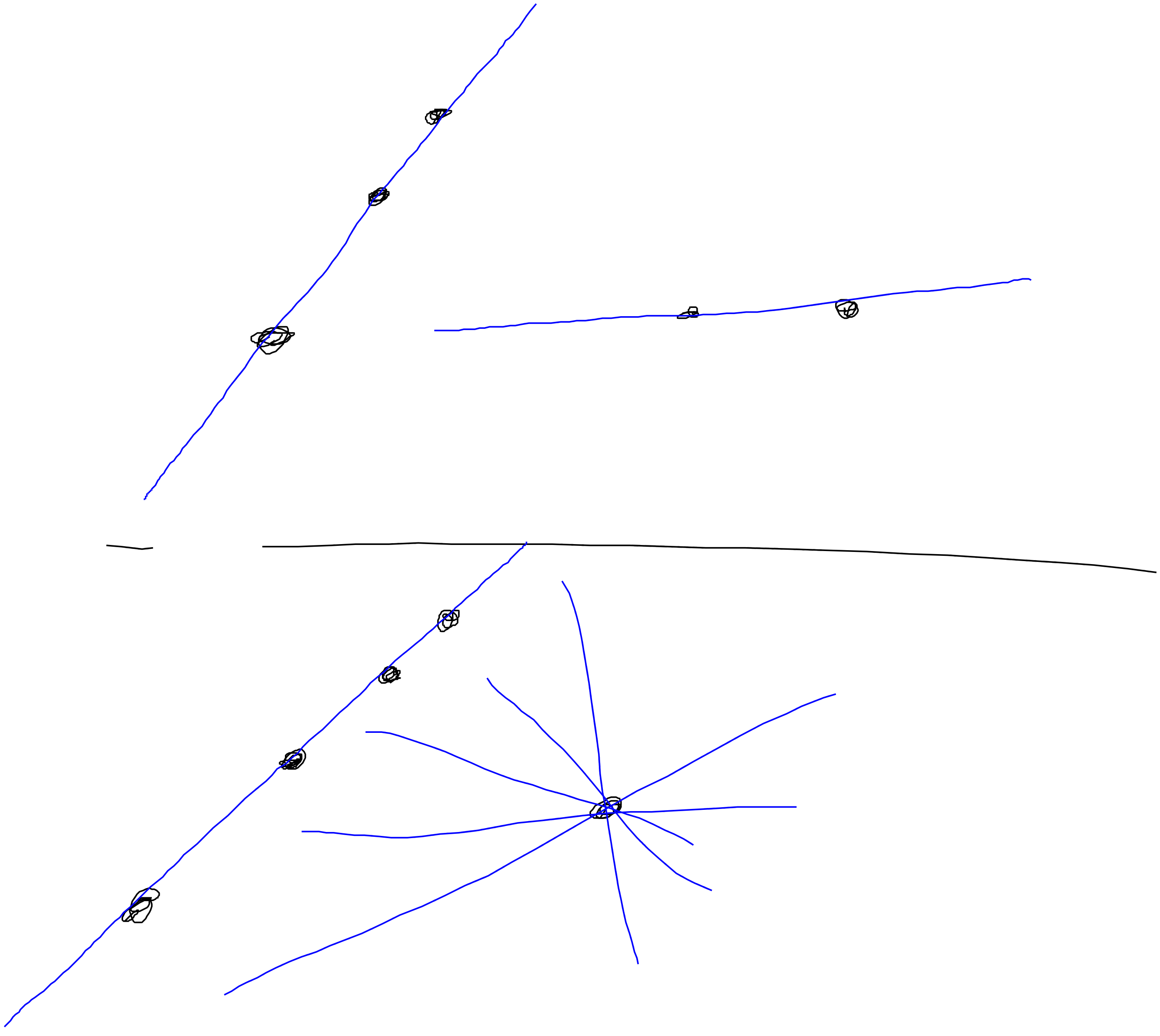
$$\lambda(2+x) + \mu y = 0$$

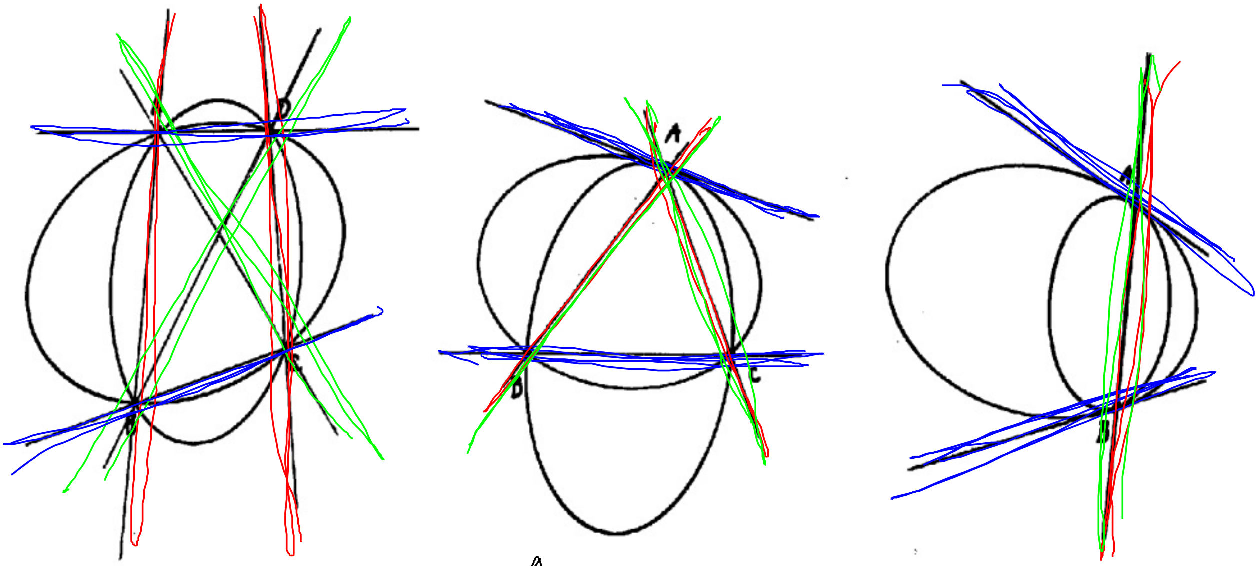
lo è

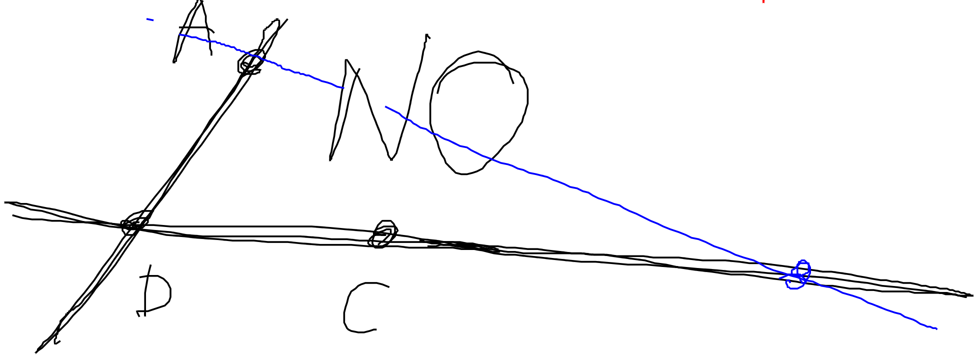
Perciò la loro intersezione

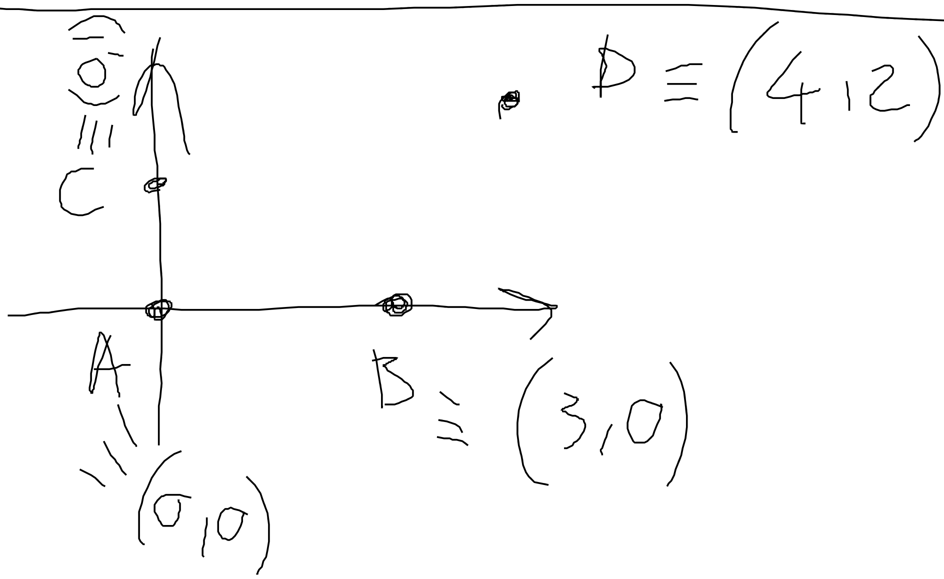
$$q : \begin{cases} 2 + x = 0 \\ y = 0 \end{cases}$$

è l'asse di rotazione.









Fascio per A, B, C, D

$\Gamma_1 = AB \cup CD \quad \Gamma_2 = AC \cup BD$

$AB: \; y = 0$

$CD: \; \dfrac{x-0}{4-0} = \dfrac{y-1}{2-1} \quad \dfrac{x}{4} = y-1$

$x - 4y + 4 = 0$

$AC: \; x = 0$

$BD: \; \dfrac{x-3}{4-3} = \dfrac{y-0}{2-0} \quad \dfrac{x-3}{1} = \dfrac{y}{2}$

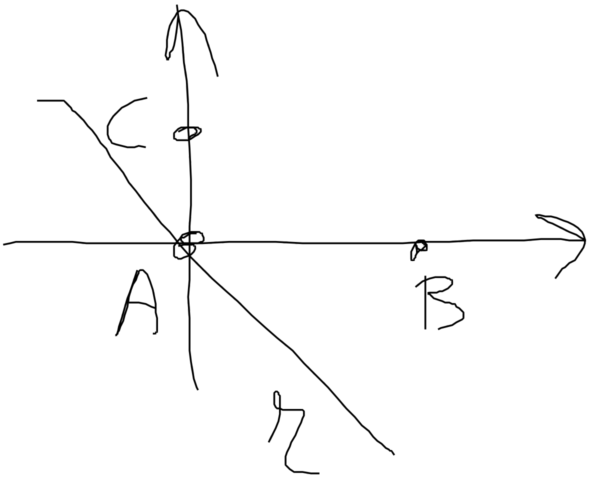
$2x - y - 6 = 0$

$\Gamma_1: \; y(x-4y+4) = 0$

$\Gamma_2: \; x(2x-y-6) = 0$

$\mathcal{F}: \; \lambda y(x-4y+4) + \mu x(2x-y-6) = 0$

Fascio di coniche per B, C e tangenti in A ad $r: x+y=0$



$\Gamma_1 = BC \cup r$

$\Gamma_2 = AB \cup AC$

$$BC: \frac{x-0}{3-0} = \frac{y-1}{0-1} \qquad \frac{x}{3} = \frac{y-1}{-1}$$

$$-x = 3y-3$$

$$x+3y-3=0$$

$$\Gamma_1: (x+y)(x+3y-3)=0$$

$$\Gamma_2: xy$$

$$\mathcal{F}: \lambda(x+y)(x+3y-3)+\mu xy=0$$