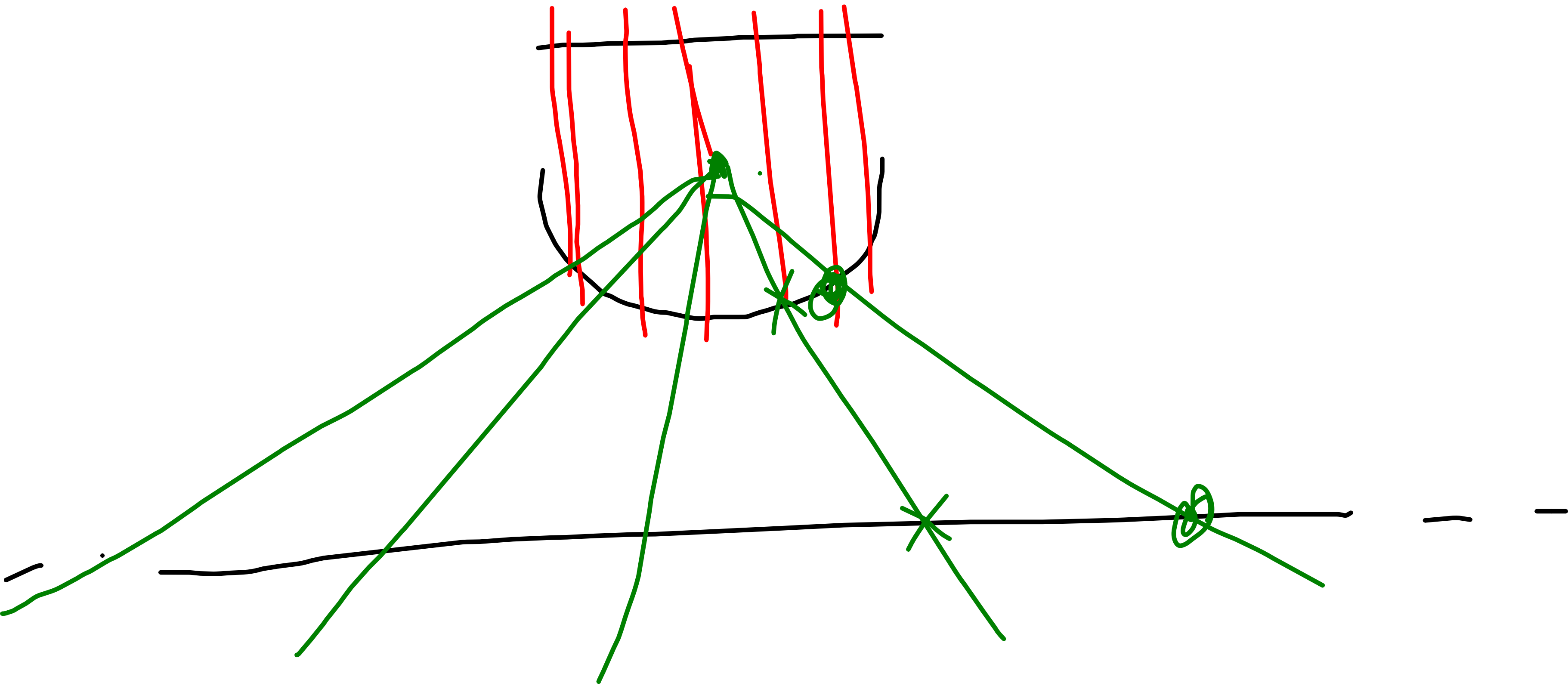


$\#\mathbb{N} \le \#\mathbb{Q}^+$

$\#\mathbb{Z} \le \#\mathbb{Q}$

$\#\mathbb{N} > \#\mathbb{Q}^+$

$f: \mathbb{N} \longrightarrow (0,1)$ $\qquad \#\mathbb{N} \cancel{>} \#(0,1)$

dimostro f non suriettiva $\qquad \aleph_0 \qquad = \#\mathbb{R}$

$\qquad \aleph_1$

$1 \longmapsto 0,a_1 a_2 a_3 a_4 a_5 \ldots$

$2 \longmapsto 0,b_1 b_2 b_3 b_4 b_5$

$3 \longmapsto 0,c_1 c_2 c_3 c_4 c_5$

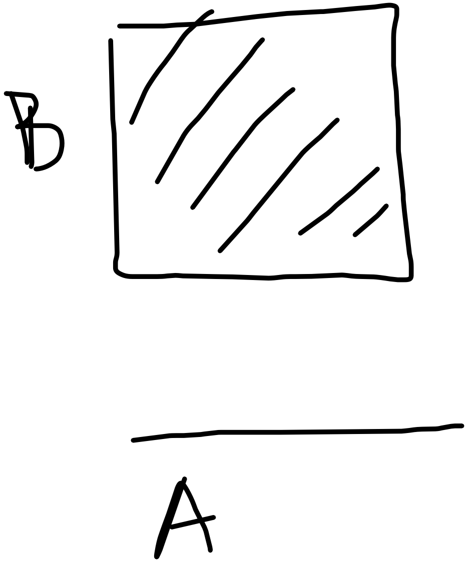
$4 \longmapsto 0,d_1 d_2 d_3 d_4 d_5$

Il numero $0,\gamma_1 \gamma_2 \gamma_3 \gamma_4 \ldots$

con

$$\gamma_1 \neq a_1$$
$$\gamma_2 \neq b_2$$
$$\gamma_3 \neq c_3$$
$$\gamma_4 \neq d_4$$
$$\vdots$$

non è immagine di alcun $n \in \mathbb{N}$



$f : A \longrightarrow B$

$0,a_1a_2a_3a_4a_5a_6 \text{---}$

$\left(0,a_1a_3a_5\cdots,0,a_2a_4a_6\cdots\right)$

$0,573\overline{9}$

$\|$

$0,574\overline{0}$

$f: X \longrightarrow Y$ biiettiva

$A, B \subset X$

$A \cap B \neq \emptyset \iff f(A) \cap f(B) \neq \emptyset$

$$a_1 x_1 + \cdots + a_n x_n + b = 0$$

$$x_1 = \frac{X_1}{X_0}, \ldots, x_n = \frac{X_n}{X_0}$$

$$a_1 \frac{X_1}{X_0} + \cdots + a_n \frac{X_n}{X_0} + b = 0$$

$0 \neq$ $\neq 0$

$X_0 \cdot$

$$a_1 X_1 + \cdots + a_n X_n + b X_0 = 0$$

$$\begin{cases} bx + cy + a = 0 \\ ex + fy + d = 0 \end{cases}$$

$$x = \frac{X_1}{X_0} \quad y = \frac{X_2}{X_0}$$

$$\begin{cases} b\frac{X_1}{X_0} + c\frac{X_2}{X_0} + a = 0 \\ e\frac{X_1}{X_0} + f\frac{X_2}{X_0} + d = 0 \end{cases}$$

$$\cdot X_0$$

$$\begin{cases} aX_0 + bX_1 + cX_2 = 0 \\ dX_0 + eX_1 + fX_2 = 0 \end{cases}$$

$r \parallel s$ $\quad r: \begin{cases} 2x-3y+1=0 \\ s: 4x-6y+3=0 \end{cases}$ $\quad \left(\begin{array}{cc|c} 2 & -3 & -1 \\ 4 & -6 & -3 \end{array}\right)$

$\rho = 1 \quad \rho = 2$

un vettore libero di r (e di s): $\vec{v} \equiv (3,2) \sim (6,4)$

$\vec{r}: 2x-3y=0$

$\vec{s}: 4x-6y=0$

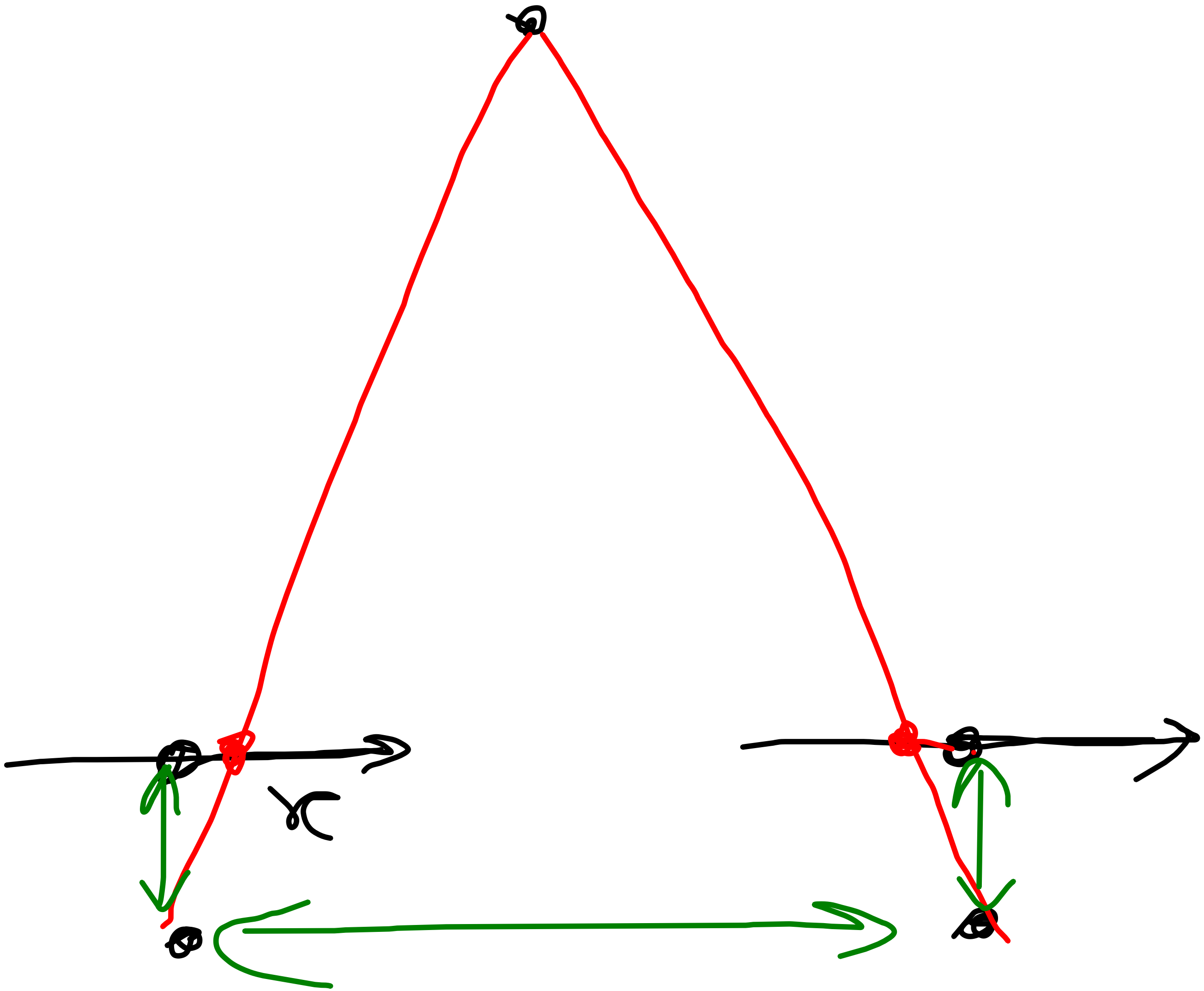
$r \cap s = \emptyset$

$$r: \begin{cases} x_0 + 2x_1 - 3x_2 = 0 \\ s: \ 3x_0 + 4x_1 - 6x_2 = 0 \end{cases} \qquad \begin{pmatrix} 1 & 2 & -3 \\ 3 & 4 & -6 \end{pmatrix}$$

$$r \cap s = \left\{ (\bar{x}_0, \bar{x}_1, \bar{x}_2) \right\}$$

$$= \left(\begin{vmatrix} 2 & -3 \\ 4 & -6 \end{vmatrix}, - \begin{vmatrix} 1 & -3 \\ 3 & -6 \end{vmatrix}, \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \right) =$$

$$= (0, -3, -2) \sim (3, 2) \sim (6, 4)$$



In $\mathbb{R}P^2$

$$3X_0^2 - 6X_0X_1 + X_2^2 + 4X_0X_2 - 8X_1X_2 = 0$$

$f =$

$$A = \begin{pmatrix} 3 & -3 & 2 \\ -3 & 0 & -4 \\ 2 & -4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X_0 & X_1 & X_2 \end{pmatrix} \cdot \begin{pmatrix} 3 & -3 & 2 \\ -3 & 0 & -4 \\ 2 & -4 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} =$$

$$= \begin{pmatrix} (3X_0-3X_1+2X_2) & (-3X_0-4X_1) & (2X_0-4X_1+X_2) \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} =$$

$$= X_0(3X_0-3X_1+2X_2)+X_1(-3X_0-4X_1)+X_2(2X_0-4X_1+X_2) =$$

$$= 3X_0^2 - 3X_0X_1 + 2X_0X_2 - 3X_1X_0 - 4X_1X_2 +$$

$$+ 2X_2X_0 - 4X_2X_1 + X_2^2 =$$

$$= 3X_0^2 - 6X_0X_1 + 4X_0X_2 - 8X_1X_2 + X_2^2$$

Sia $\bar{P} = (\bar{x}_0, \dots, \bar{x}_n) \in W[f]$

$$W[f]: \quad A \cdot \begin{pmatrix} x_0 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$I_n[f] \cdot (x_0 \ - \ x_n) \cdot A \cdot \begin{pmatrix} x_0 \\ \vdots \\ x_n \end{pmatrix} = 0$$

$$A \cdot \begin{pmatrix} \overline{x}_a \\ \vdots \\ \overline{x}_n \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\Rightarrow \left(\overline{x}_a \cdot \ldots \cdot \overline{x}_n \right) \cdot A \cdot \begin{pmatrix} \overline{x}_a \\ \vdots \\ \overline{x}_n \end{pmatrix} =$$

$$= (\bar{x}_0 \cdots \bar{x}_n) \cdot \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix} = 0$$

cioè

$$\bar{P} \in W[f] \Rightarrow \bar{P} \in \mathrm{Im}[f]$$