$$\begin{cases} X_0^2 + X_1^2 - X_2^2 - X_3^2 = 0 \\ X_0 = \alpha \\ X_1 = \beta \\ X_2 = \gamma \\ X_3 = \pm \dfrac{a\alpha + b\beta - c\gamma}{\sqrt{a^2+b^2-c^2}} \end{cases}$$

$$\frac{1}{a^2+b^2-c^2}\begin{pmatrix} (b^2-c^2) & -ab & ac \\ -ab & (a^2-c^2) & bc \\ ac & bc & -(a^2+b^2) \end{pmatrix}$$

$$-\left(a^2\alpha^2 + b^2\beta^2 + c^2\gamma^2 + 2a\alpha b\beta - 2a\alpha c\gamma - 2b\beta c\gamma\right)$$

$$\alpha^2 + \beta^2 + \gamma^2 - \frac{(a\alpha + b\beta - c\gamma)^2}{a^2+b^2-c^2}$$

$$\alpha^2\left(1 - \frac{a^2}{a^2+b^2-c^2}\right) + \beta^2\left(1 - \frac{b^2}{}\right) - \gamma^2\left(1 + \frac{c^2}{}\right) - 2\alpha\beta ab + 2\alpha\gamma ac + 2\beta\gamma bc$$

$$x^2+3y^2-2xy-4x=0 \qquad A=\begin{pmatrix}0 & -2 & 0\\ -2 & 1 & -1\\ 0 & -1 & 3\end{pmatrix}$$

Cerco il centro come intersez. delle polari di X_∞ e Y_∞

$$\begin{cases}(0\ 1\ 0)\begin{pmatrix}0 & -2 & 0\\ -2 & 1 & -1\\ 0 & -1 & 3\end{pmatrix}\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0\\ (0\ 0\ 1)\begin{pmatrix}0 & -2 & 0\\ -2 & 1 & -1\\ 0 & -1 & 3\end{pmatrix}\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0\end{cases}$$

$$\begin{pmatrix}X_0\\ X_1\\ X_2\end{pmatrix}=0 \qquad \begin{pmatrix}-2 & 1 & -1\\ 0 & -1 & 3\end{pmatrix}$$

$$\begin{cases}-2X_0+X_1-X_2=0\\ -X_1+3X_2=0\end{cases}$$

$$(X_0,X_1,X_2)\sim\left(\begin{vmatrix}1 & -1\\ -1 & 3\end{vmatrix}, -\begin{vmatrix}-2 & -1\\ 0 & 3\end{vmatrix}, \begin{vmatrix}-2 & 1\\ 0 & -1\end{vmatrix}\right)= \quad =(A_0^0, A_1^0, A_2^0)$$

$$\begin{cases}(-2\ 1\ -1)\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0\\ (0\ -1\ 3)\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0\end{cases} \qquad \begin{cases}-2+x-y=0\\ -x+3y=0\end{cases} \qquad \begin{cases}x-y=2\\ 2y=2\end{cases} \quad (2,6,2) \quad (1,3,1)$$

$$\begin{cases}x=3\\ y=1\end{cases}$$

$P_\infty \equiv (0, 2, 5)$ Facciamone la polare

$$\underset{1\times 3}{\begin{pmatrix} 0 & 2 & 5 \end{pmatrix}} \underset{3\times 3}{\begin{pmatrix} 0 & -2 & 0 \\ -2 & 1 & -1 \\ 0 & -1 & 3 \end{pmatrix}} \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$$

1×3

$$-4 - 3x + 13y = 0$$

$$\quad\quad 3 \quad\quad 1$$

$$-4 - 9 + 13 = 0$$

$$x^2-6xy+9y^2-4x+1=0 \qquad A=\begin{pmatrix}1&-2&0\\-2&1&-3\\0&-3&9\end{pmatrix}$$

$$|A|=\begin{vmatrix}1&0&0\\-2&-3&-3\\0&-3&9\end{vmatrix}\neq 0 \qquad A^0_0=\begin{vmatrix}1&-3\\-3&9\end{vmatrix}=0$$

parabala

$$\begin{matrix}X_\infty\\Y_\infty\end{matrix}\left\{\begin{matrix}(0\ 1\ 0)\\(0\ 0\ 1)\end{matrix}\right. \qquad \begin{pmatrix}1&-2&0\\-2&1&-3\\0&-3&9\end{pmatrix}\begin{pmatrix}X_0\\X_1\\X_2\end{pmatrix}=0 \quad \begin{cases}-2X_0+X_1-3X_2=0\\ \quad -3X_1+9X_2=0\end{cases}$$

$$\begin{pmatrix}-2&1&-3\\0&-3&9\end{pmatrix} \quad (X_0,X_1,X_2)\sim\left(\begin{vmatrix}1&-3\\-3&9\end{vmatrix},-\begin{vmatrix}-2&-3\\0&9\end{vmatrix},\begin{vmatrix}-2&1\\0&-3\end{vmatrix}\right)=$$

$$=(0,18,6)\sim(0,3,1)$$

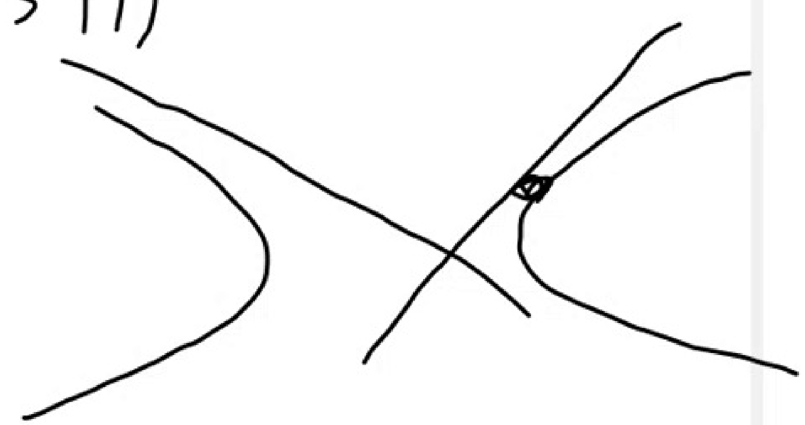
$P_\infty = (0, 2, 5)$

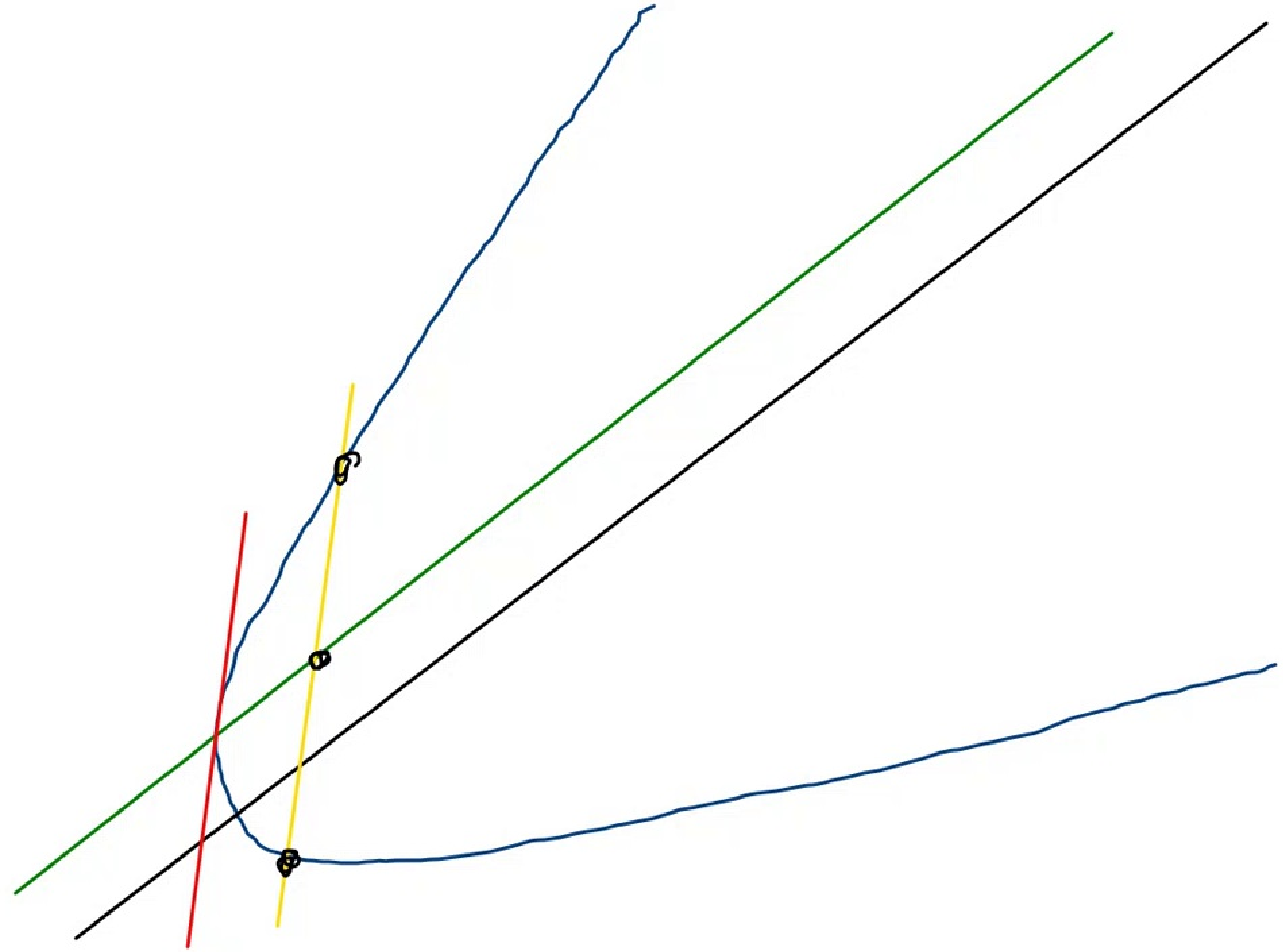
polare:

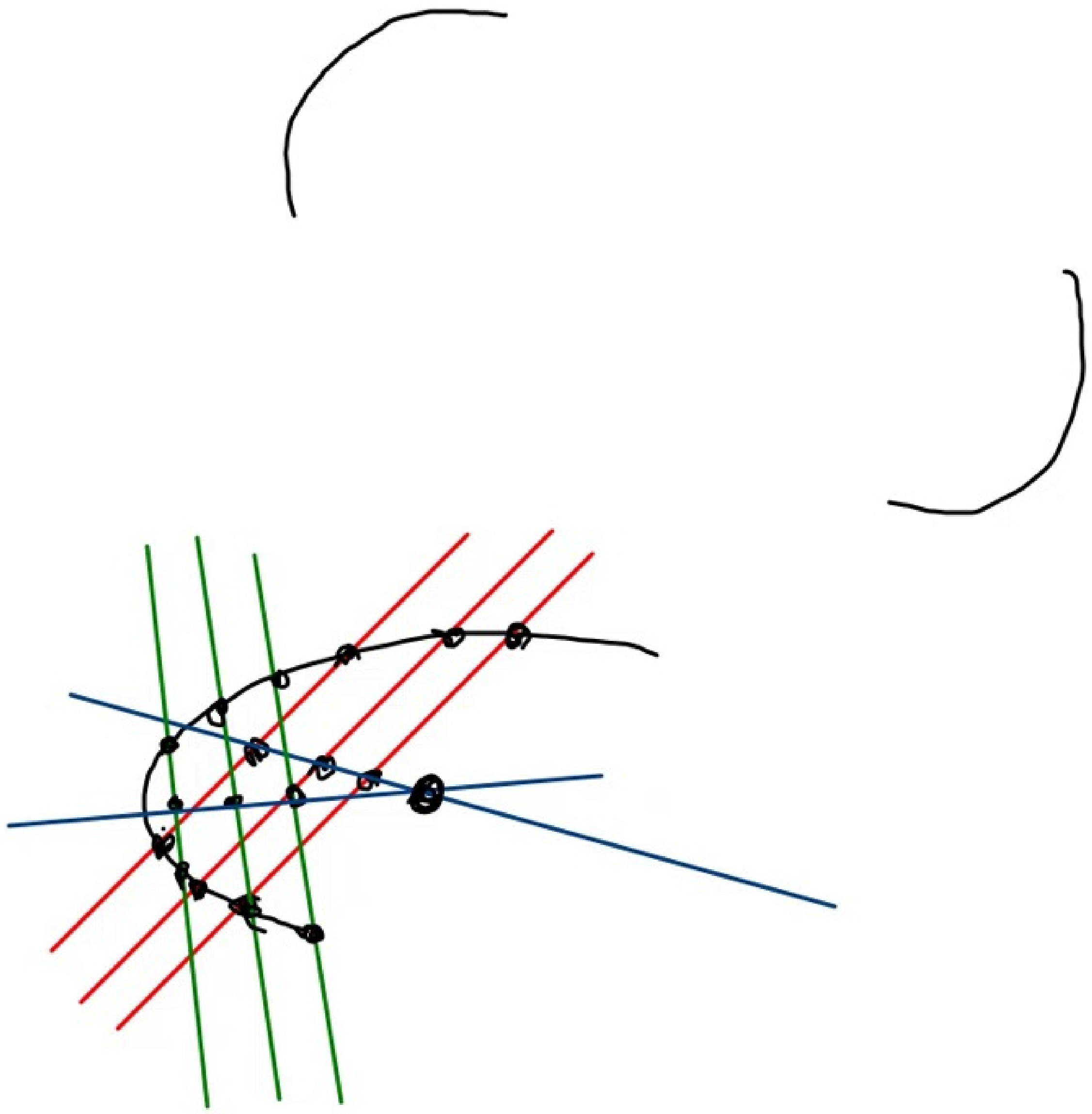
$$(0\ 2\ 5)\begin{pmatrix} 1 & -2 & 0 \\ -2 & 1 & -3 \\ 0 & -3 & 9 \end{pmatrix}\begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$$

$-4 - 13x + 39y = 0$ è un diametro

$(l, m) \sim (39, 13) \sim (3, 1)$







$[f]$ con discrim. $A = \begin{pmatrix} a_0^0 & \cdots & \\ & \cdots & a_n^n \end{pmatrix}$

Iperpiano diametrale generico: polare del generico punto improprio $P_\infty = (0, l_1, \dots, l_n)$

$$\tau(P_\infty) : \begin{pmatrix} X_0 & X_1 & \cdots & X_n \end{pmatrix} \begin{pmatrix} a_0^0 & \cdots & & a_n^0 \\ a_0^1 & \boxed{a_1^1} & \cdots & a_n^1 \\ & & \cdots & \\ a_0^n & a_1^n & \cdots & a_n^n \end{pmatrix} \begin{pmatrix} 0 \\ l_1 \\ \vdots \\ l_n \end{pmatrix} = 0$$

$$\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = M_{00} \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

$$\begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_n \end{pmatrix} = \begin{pmatrix} \cancel{a_0^0 0} + a_1^0 l_1 + \cdots + a_n^0 l_n \\ \cancel{a_0^1 0} + a_1^1 l_1 + \cdots + a_n^1 l_n \\ \vdots \\ \cancel{a_0^n 0} + a_1^n l_1 + \cdots + a_n^n l_n \end{pmatrix}$$

$$\tau(P_\infty) : b_0 X_0 + b_1 X_1 + \cdots + b_n X_n = 0$$
$$b_1 x_1 + \cdots + b_n x_n + b_0 = 0$$

Retta r di coeff. dir. $l_1, \dots, l_n$

Iperpiano Π: $b_1 x_1 + \dots + b_n x_n + b_0 = 0$

rispetto a un rif. cartesiano

$$r \perp \Pi \iff (b_1, \dots, b_n) \sim (l_1, \dots, l_n)$$

$$\iff \exists \lambda \neq 0 \text{ t.c. } (b_1, \dots, b_n) = \lambda \cdot (l_1, \dots, l_n)$$

Perciò $\tau(P_\infty)$ è un iperp. principale $\iff \exists \lambda \neq 0$ t.c. $\begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$

$$\iff \exists \lambda \neq 0 \text{ t.c. } M_{00}' \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix} = \lambda \begin{pmatrix} l_1 \\ \vdots \\ l_n \end{pmatrix}$$

$$x^2+6xy+9y^2-20x+1=0$$

a) classificare b) asse

$$A=\begin{pmatrix} 1 & -10 & 0 \\ -10 & \boxed{1} & \boxed{3} \\ 0 & \boxed{3} & \boxed{9} \end{pmatrix}$$

a) $|A| = \begin{vmatrix} 1 & 0 & 0 \\ -10 & -99 & 3 \\ 0 & 3 & 9 \end{vmatrix} \neq 0$ non deg

$M_{00} = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$ $|M_{00}| = 0$ parabola ✓

b) $\begin{vmatrix} (1-\lambda) & 3 \\ 3 & (9-\lambda) \end{vmatrix} = \cancel{9} - \lambda - 9\lambda + \lambda^2 - \cancel{9} = \lambda(\lambda - 10)$

$\lambda = 10$ $\begin{pmatrix} \cancel{-9} & \cancel{3} \\ 3 & -1 \end{pmatrix} \begin{pmatrix} l \\ m \end{pmatrix} = \begin{pmatrix} \cancel{0} \\ 0 \end{pmatrix}$ $3l - m = 0$ $(l, m) \sim (1, 3)$

$(0\ 1\ 3) \cdot \begin{pmatrix} 1 & -10 & 0 \\ -10 & 1 & 3 \\ 0 & 3 & 9 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$ $-10 + 10x + 30y = 0$

$x + 3y - 1 = 0$

$$x^2+2y^2+3z^2+5=0$$

$$A=\begin{pmatrix}5 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 3\end{pmatrix}$$

$$M^0_a=\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{pmatrix}$$

autovalori $\det\begin{pmatrix}(1-\lambda) & 0 & 0\\ 0 & (2-\lambda) & 0\\ 0 & 0 & (3-\lambda)\end{pmatrix}=(1-\lambda)(2-\lambda)(3-\lambda)$

autoval. $1, 2, 3$

$(0\,1\,0\,0)\cdot A\cdot\begin{pmatrix}1\\x\\y\\z\end{pmatrix}=0 \quad \boxed{x=0}$

$\lambda=1 \quad \begin{pmatrix}\cancel{0} & \cancel{0} & \cancel{0}\\ 0 & 1 & 0\\ 0 & 0 & 2\end{pmatrix}\begin{pmatrix}l\\m\\n\end{pmatrix}=\begin{pmatrix}\cancel{0}\\0\\0\end{pmatrix}\quad \begin{cases}m=0\\ 2n=0\end{cases} \quad P_{100}\equiv(0,1,0,0)$

$\lambda=2 \quad \begin{pmatrix}-1 & 0 & 0\\ \cancel{0} & \cancel{0} & \cancel{0}\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}l\\m\\n\end{pmatrix}=\begin{pmatrix}0\\\cancel{0}\\0\end{pmatrix}\quad \begin{cases}-l=0\\ n=0\end{cases} \quad P_{200}\equiv(0,0,1,0)$

$2y=0 \quad \boxed{y=0}$

$\lambda=3 \quad \begin{pmatrix}-2 & 0 & 0\\ 0 & -1 & 0\\ \cancel{0} & \cancel{0} & \cancel{0}\end{pmatrix}\begin{pmatrix}l\\m\\n\end{pmatrix}=\begin{pmatrix}0\\0\\\cancel{0}\end{pmatrix}\quad \begin{cases}-2l=0\\ -m=0\end{cases}$

$P_{300}\equiv(0,0,0,1)$

$3z=0 \quad \boxed{z=0}$

$$x^2+2y^2+2z^2+5=0 \qquad A=\begin{pmatrix}5 & & & \\ & 1 & & \\ & & 2 & \\ & & & 2\end{pmatrix}$$

$$M_0^0=\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{pmatrix} \qquad \begin{vmatrix}(1-\lambda) & 0 & 0\\ 0 & (2-\lambda) & 0\\ 0 & 0 & (2-\lambda)\end{vmatrix} = (1-\lambda)(2-\lambda)^2$$

$$\lambda=1 \quad \begin{pmatrix}\cancel{0} & \cancel{0} & \cancel{0}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}l\\ m\\ n\end{pmatrix}=\begin{pmatrix}\cancel{0}\\ 0\\ 0\end{pmatrix} \quad \begin{cases}m=0\\ n=0\end{cases} \quad P_{1\infty}\equiv(0,1,0,0) \quad \boxed{x=0}$$

$$\alpha\neq 0$$

$$\lambda=2 \quad \begin{pmatrix}-1 & 0 & 0\\ \cancel{0} & \cancel{0} & \cancel{0}\\ \cancel{0} & \cancel{0} & \cancel{0}\end{pmatrix}\begin{pmatrix}l\\ m\\ n\end{pmatrix}=\begin{pmatrix}0\\ \cancel{0}\\ \cancel{0}\end{pmatrix} \quad l=0 \quad P_{2\infty}\equiv(0,0,\alpha,\beta)$$

$$(\alpha,\beta)\neq(0,0)$$

$$\begin{pmatrix}0 & 0 & \alpha & \beta\end{pmatrix}\begin{pmatrix}5 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 2\end{pmatrix}\cdot\begin{pmatrix}1\\ x\\ y\\ z\end{pmatrix}=0 \qquad \begin{pmatrix}0 & 0 & 2\alpha & 2\beta\end{pmatrix}\begin{pmatrix}1\\ x\\ y\\ z\end{pmatrix}=0 \qquad 2\alpha y+2\beta z=0$$

$$\alpha y+\beta z=0$$

tutti contengono la retta $\begin{cases}y=0\\ z=0\end{cases}$

$$x^2+y^2+z^2+5=0$$

$$\begin{pmatrix} 5 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

autoval di M_a^0: 1

$$\det\begin{pmatrix} (\lambda-1) & & 0 \\ & (\lambda-1) & \\ 0 & & (\lambda-1) \end{pmatrix} = (\lambda-1)^3$$

$$(o\ l\ m\ n)\, A_1 \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$$

$$lx+my+nz=0$$

$$\lambda=1 \quad \begin{pmatrix} 0&0&0 \\ 0&0&0 \\ 0&0&0 \end{pmatrix}\begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

OGNI (o,l,m,n) è polo di un piano principale

tutti questi piani contengono il punto $(1,0,0,0)$ $(0,0,0)$

$$\begin{cases} x=0 \\ y=0 \\ z=0 \end{cases}$$

$$x^2+2y^2+3z^2-5=0$$

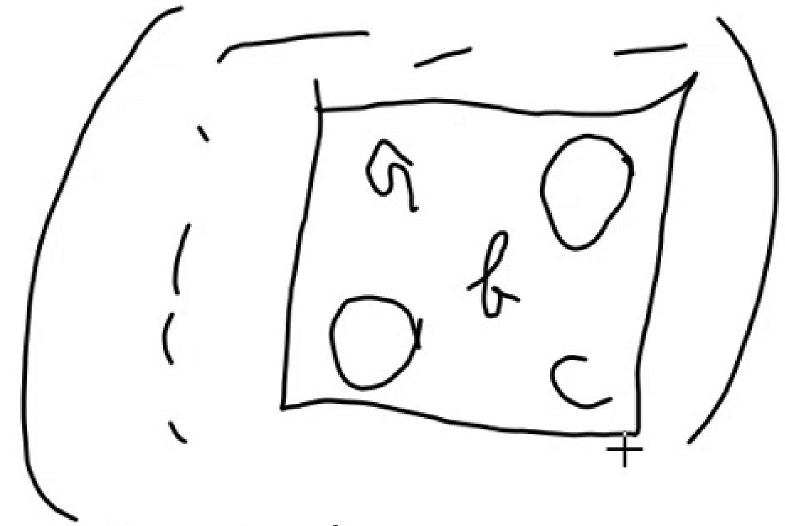
$$x^2+2y^2+2z^2-5=0$$

$$x^2+y^2+z^2-5=0$$

$$x^2+2y^2+3z^2-5=0$$

$$x^2+2y^2+2z^2-5=0$$

$$x^2+y^2+z^2-5=0$$



paraboloidi: $c=0$

iperb: a, b segno discorde

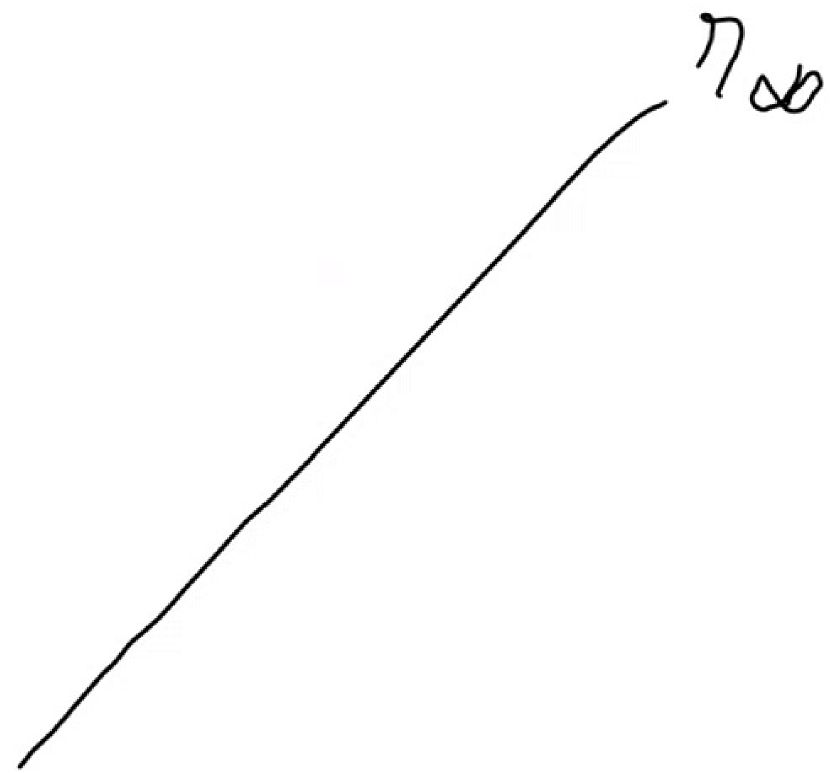
ellittico: a, b segno concorde

iperboloidi: a, b concordi ma discordi da c

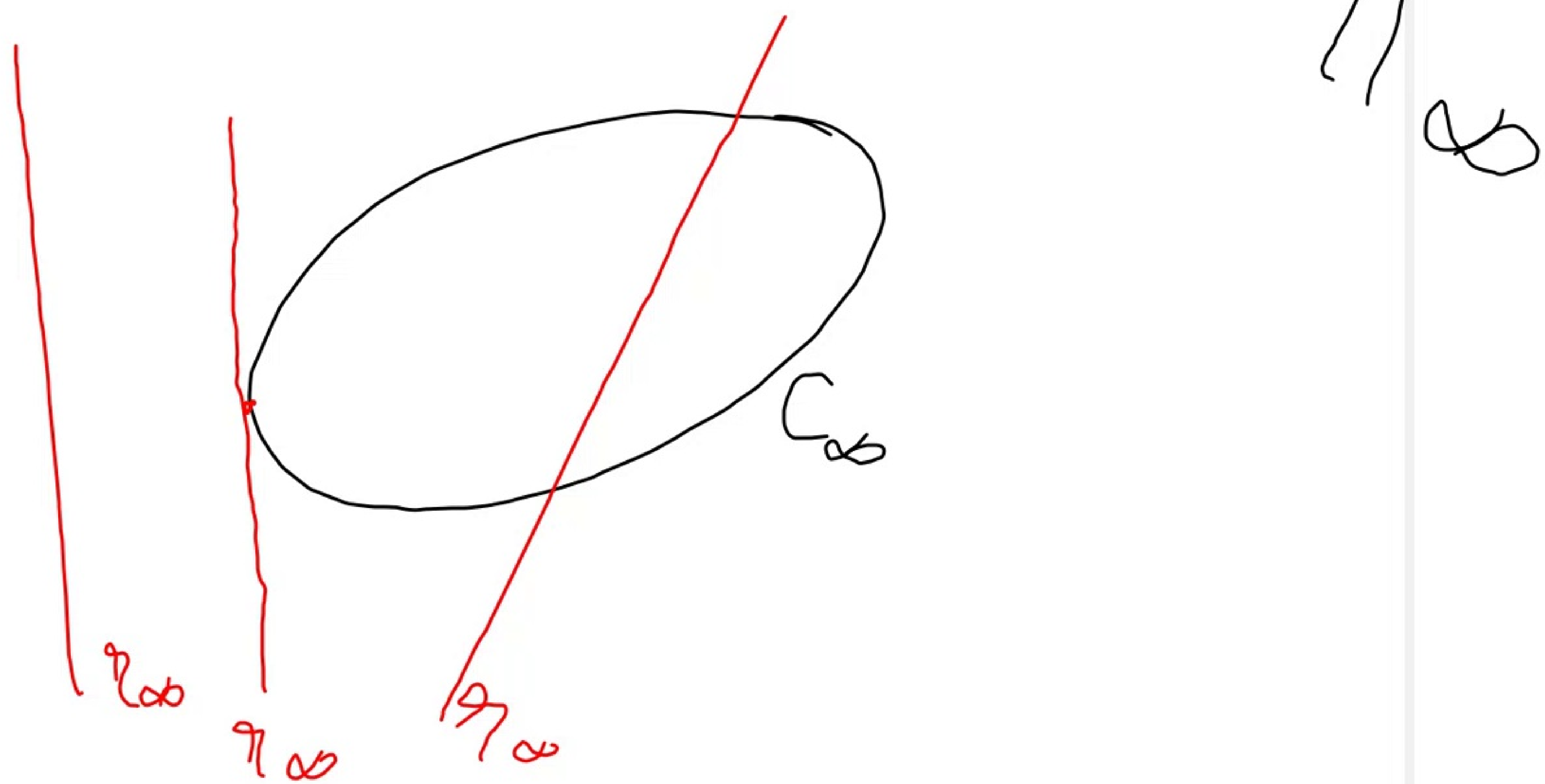
ellissoidi: a, b, c concordi

Le possibili sezioni piane di una quadrica $\mathcal{Q}$ $\pi \cap \mathcal{Q}$ sono determinate dalla intersezione della retta impropria r_∞ del piano π con la conica impropria C_∞ della quadrica $\mathcal{Q}$.

C_∞ di un ellissoide: $\emptyset$



C_∞ di un iperboloide



paraboloide ellittico

paraboloide iperbolico