Trovare i piani principali della quadrica $x^2+y^2+9z^2-4xy+2=0$

$$A=\begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 9 \end{pmatrix}$$

$P_\infty \equiv (0, l, m, n)$ $\qquad (0\ l\ m\ n)\, A \cdot \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$

polare di P_∞: $\quad \big(0\ \ \underbrace{(l-2m)}_{a=}\ \ \underbrace{(-2l+m)}_{b=}\ \ \underbrace{9n}_{c=}\big) \cdot \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$

$$\begin{pmatrix} l \\ m \\ n \end{pmatrix} \sim \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} l-2m \\ -2l+m \\ 9n \end{pmatrix}$$

$$\begin{cases} l(l-2m)+m(m-2l)=0 \\ 9ln+n(2m-l)=0 \end{cases}$$

Trovare, se c'è, il piano su cui giace la curva C:
$$\begin{cases} x = u^3 + u \\ y = 2u^3 + u^2 \\ z = u^2 - 2u + 2 \end{cases}$$

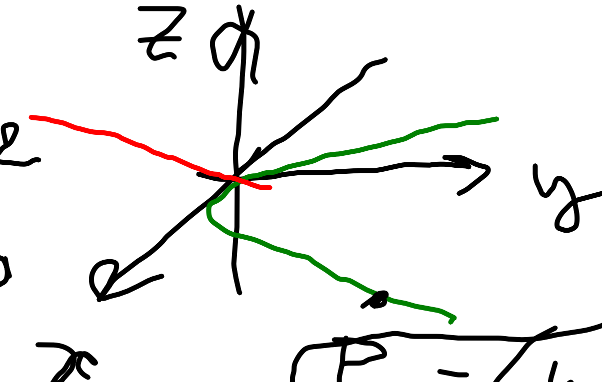
Generico piano $ax + by + cz + d = 0$

$$\Phi(u) = a(u^3+u) + b(2u^3+u^2) + c(u^2-2u+2) + d =$$
$$= u^3(a+2b) + u^2(b+c) + u(a-2c) + 2c + d$$

identi. nullo $\Longleftrightarrow$ $\begin{cases} a+2b=0 \\ b+c=0 \\ a-2c=0 \\ 2c+d=0 \end{cases}$ $\begin{cases} a+2b=0 \\ b+c=0 \\ -2b-2c=0 \\ 2c+d=0 \end{cases}$ $\begin{cases} \text{—} \\ \text{—} \\ 0=0 \\ \text{—} \end{cases}$

$\begin{cases} a = 2\alpha \\ b = -\alpha \\ c = \alpha \\ d = -2\alpha \end{cases}$ scelgo $\alpha = 1$ $\boxed{2x - y + z - 2 = 0}$

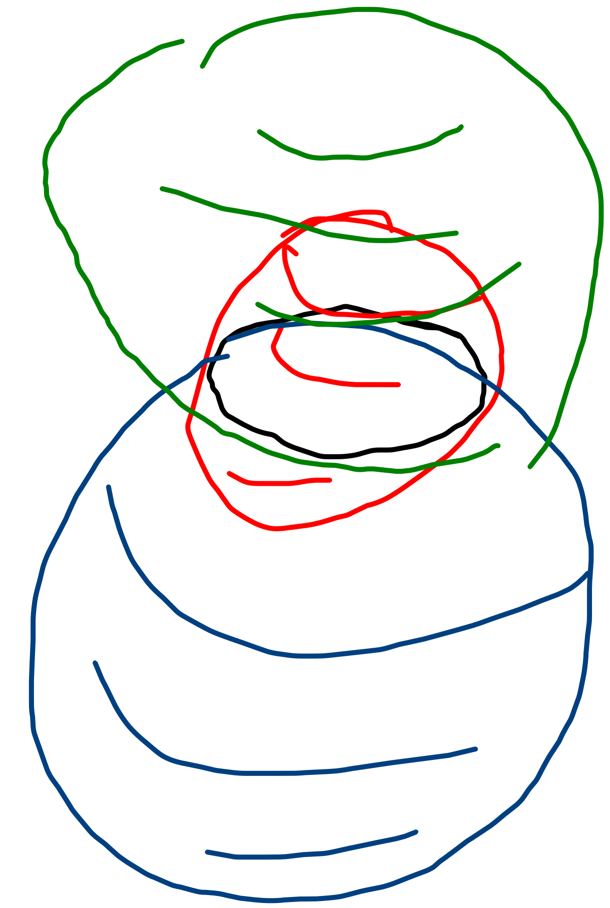
Es. 10 $\quad C : \begin{cases} z = 0 \\ y = x^2 \end{cases} \quad \begin{cases} x = u \\ y = u^2 \\ z = 0 \end{cases}$



$P_u \equiv (u, u^2, 0)$

a) spf. di rot. attorno all'a retta $r : \begin{cases} x = z \\ y = 0 \end{cases}$

$(l, m, n) \sim (1, 0, 1)$



Π_u per P_u, $\perp r$

$$1(x-u) + 0(y-u^2) + 1(z-0) = 0$$

Σ_u sfera con centro O, per P_u

$$\Sigma_u : (x-0)^2 + (y-0)^2 + (z-0)^2 = (u-0)^2 + (u^2-0)^2 + (0-0)^2$$

$$x^2 + y^2 + z^2 = u^2 + u^4$$

$$\begin{matrix} \Pi_u \\ \Sigma_u \end{matrix} \begin{cases} x + z - u = 0 \qquad u = x + z \\ x^2 + y^2 + z^2 = u^2 + u^4 \end{cases}$$

$$\mathcal{S} : x^2 + y^2 + z^2 = (x+z)^2 + (x+z)^4$$

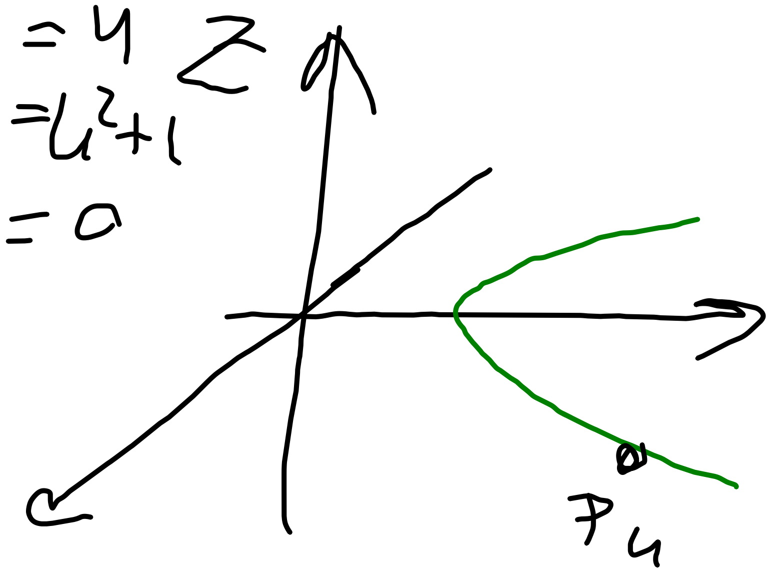
spf di rot. attorno all'asse z,

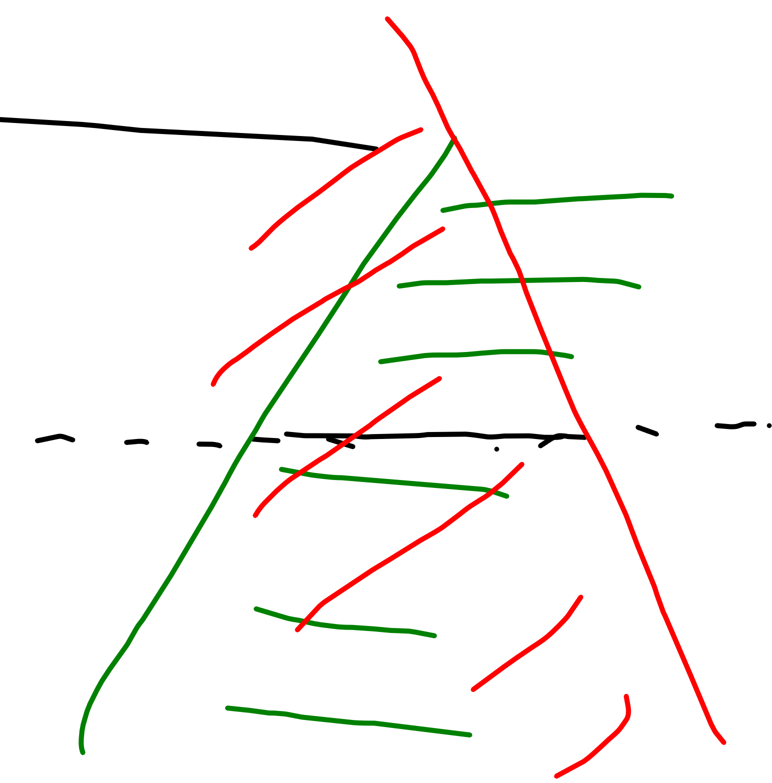
Π_u per P_u, $\perp$ asse z : $\Pi_u : z = 0$

Σ_u sfera per P_u, con centro O

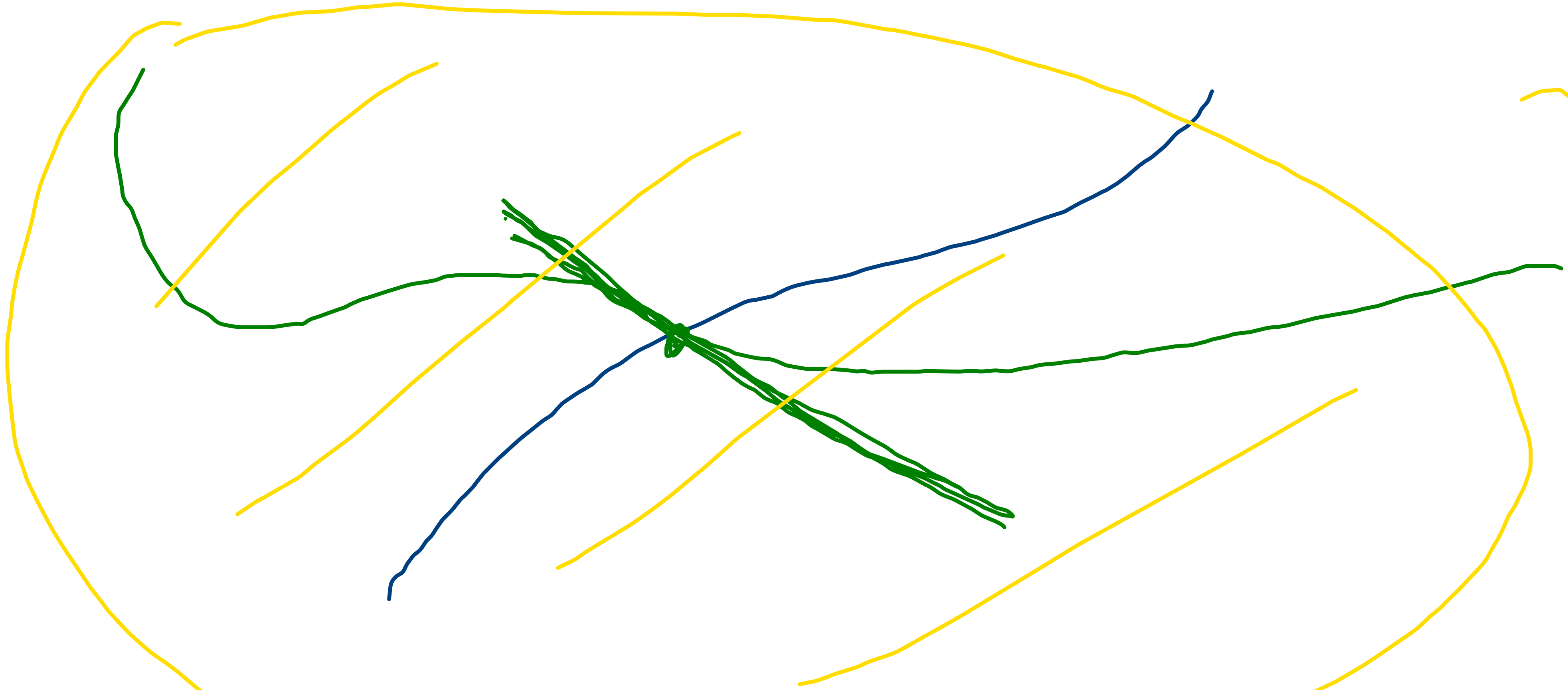
$$\begin{cases} x^2+y^2+z^2 = u^2+u^4 \\ z = 0 \end{cases} \longrightarrow z = 0$$

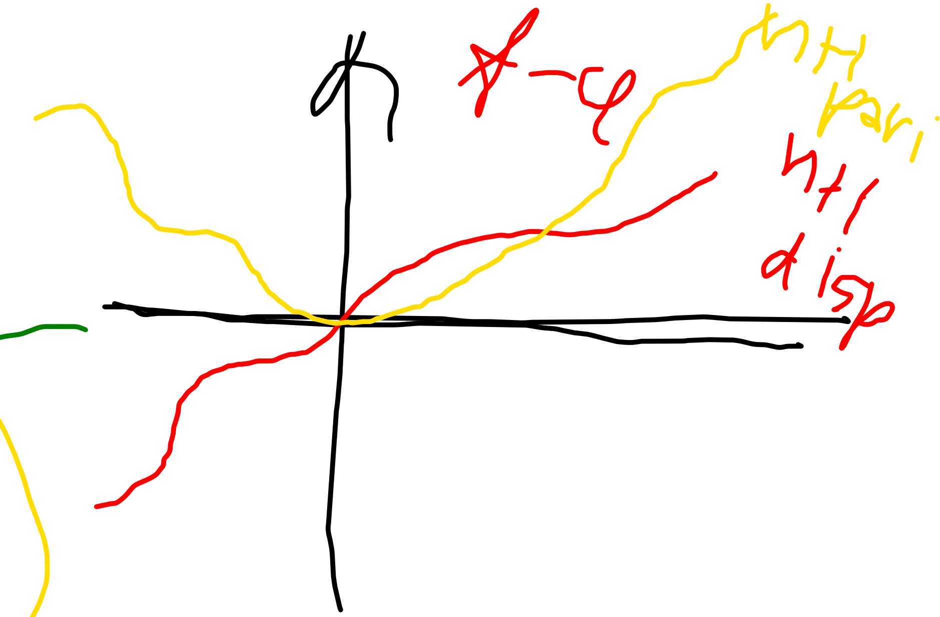
$$C : \begin{cases} z = 0 \\ y = x^2+1 \end{cases} \qquad \begin{cases} x = u \\ y = u^2+1 \\ z = 0 \end{cases}$$





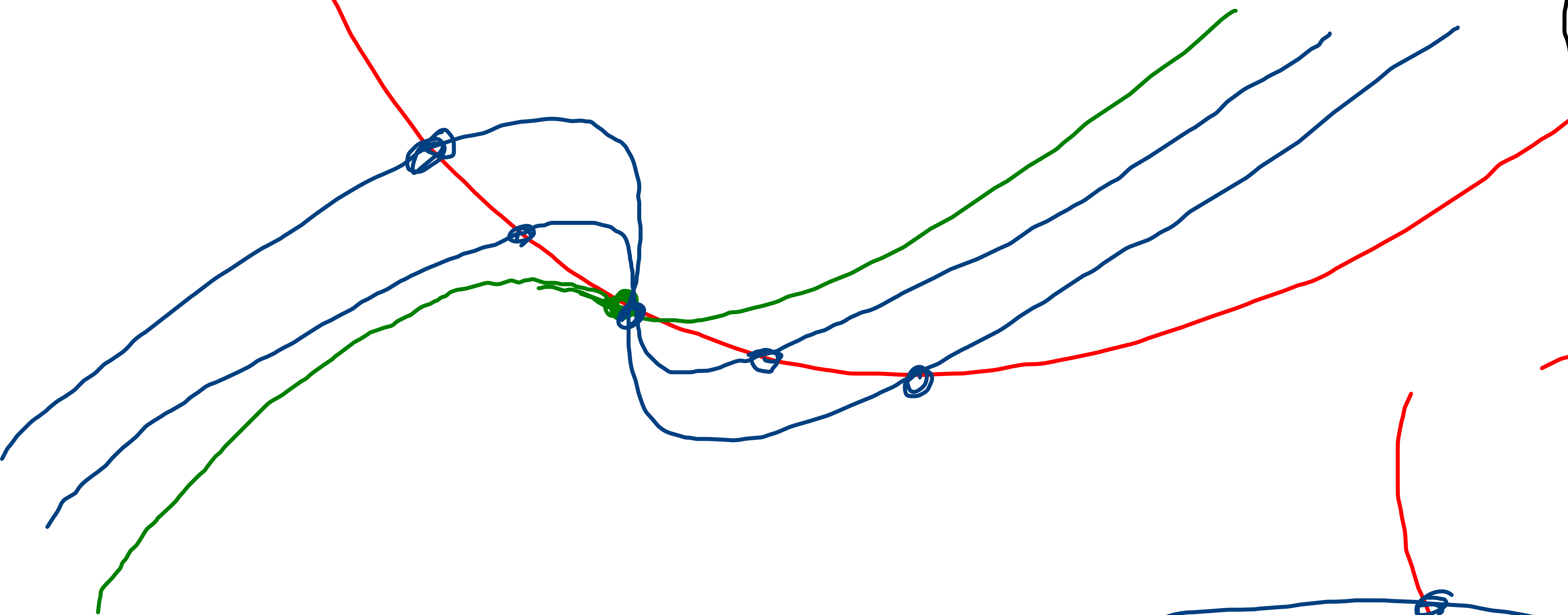
$$\Sigma_u : \begin{cases} x^2+y^2+z^2 = u^2+(u^2+1)^2 \\ z = 0 \end{cases}$$

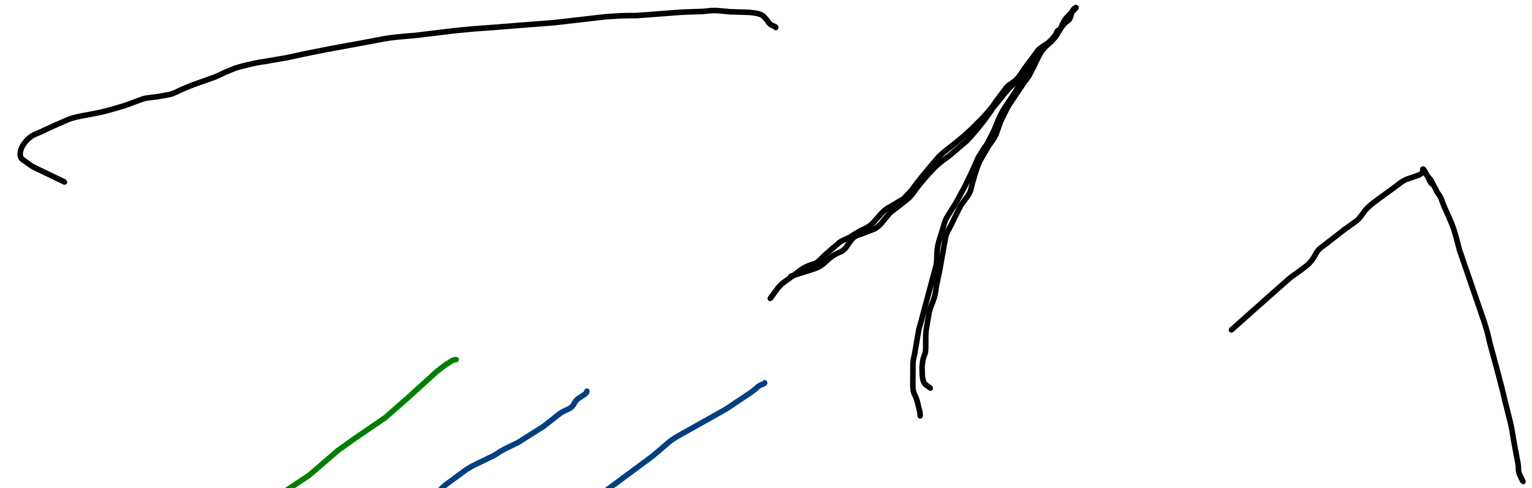


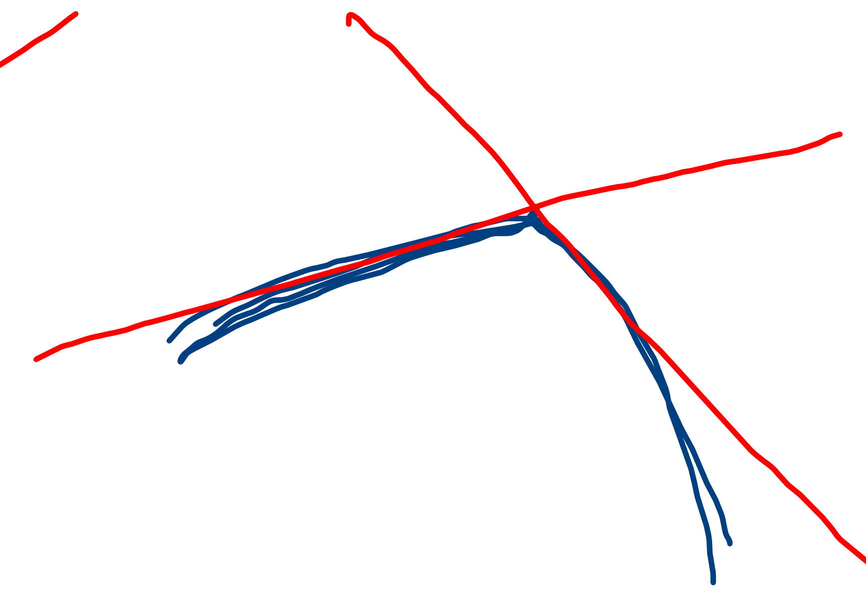


$$f(x) = \cancel{f'(0)x} + \cancel{\frac{1}{2} f''(0)x^2} + \cdots + \cancel{\frac{1}{n!} f^{(n)}(0)x^n} + \frac{1}{(n+1)!} f^{(n+1)}(0)x^{n+1} +$$

$$-\varphi(x) = \cancel{\varphi'(0)x} + \cancel{\frac{1}{2} \varphi''(0)x^2} + \cdots + \cancel{\frac{1}{n!} \varphi^{(n)}(0)x^n} + \frac{1}{(n+1)!} \varphi^{(n+1)}(0)x^{n+1} +$$



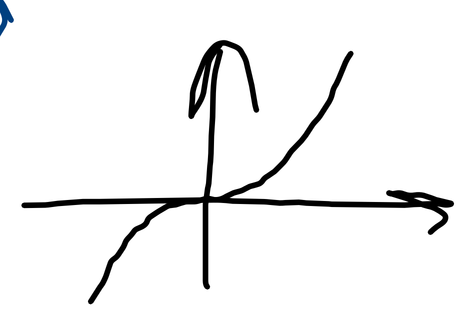


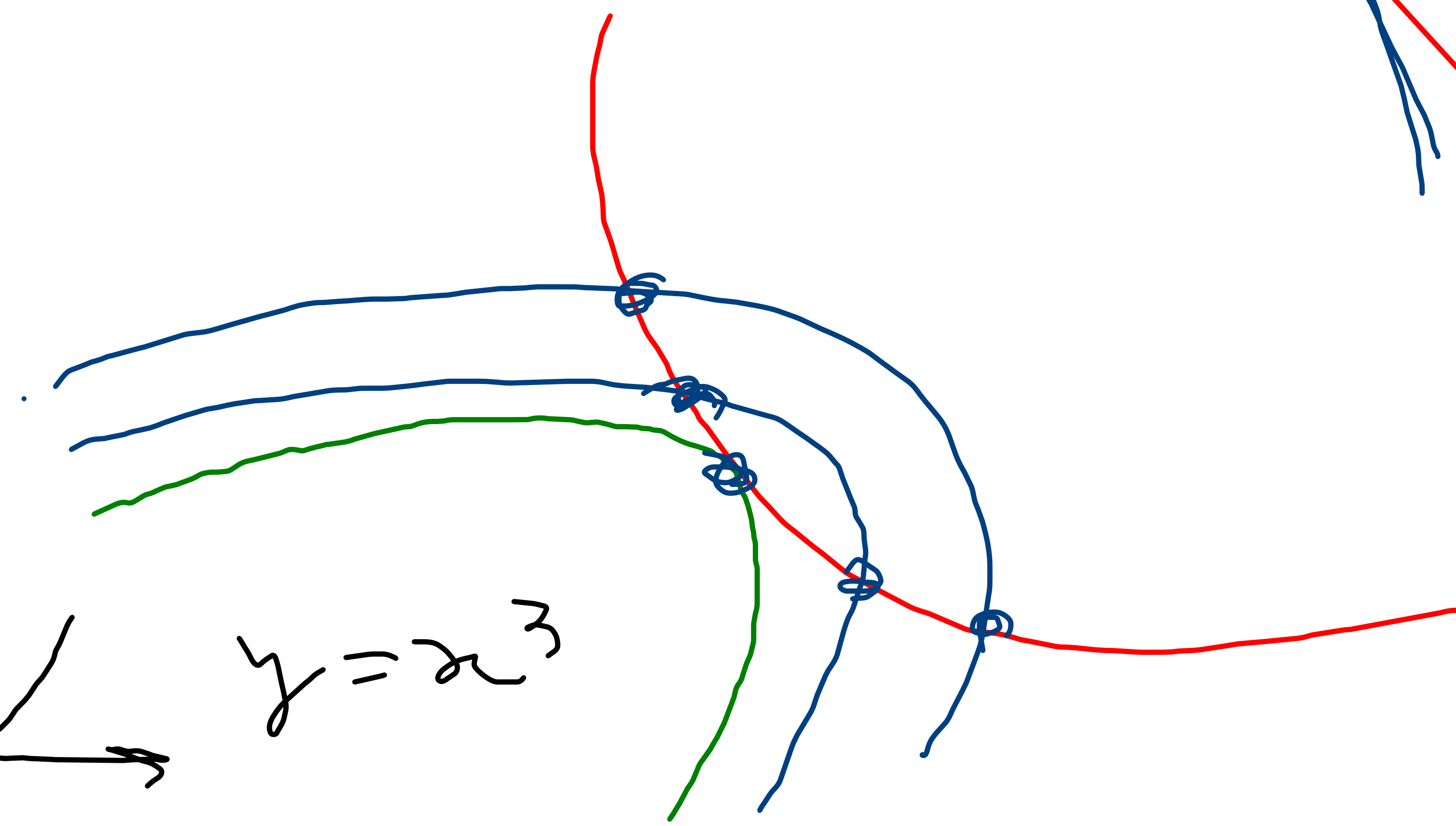


8,97

$x^2 - 6x + 9 = 0$

10,3



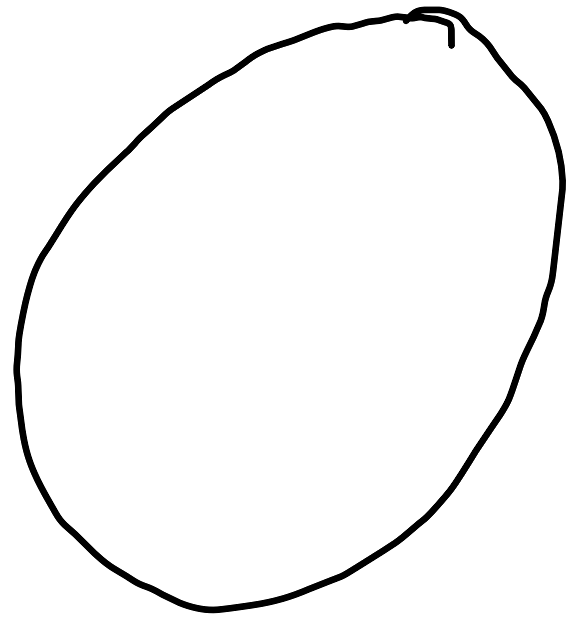


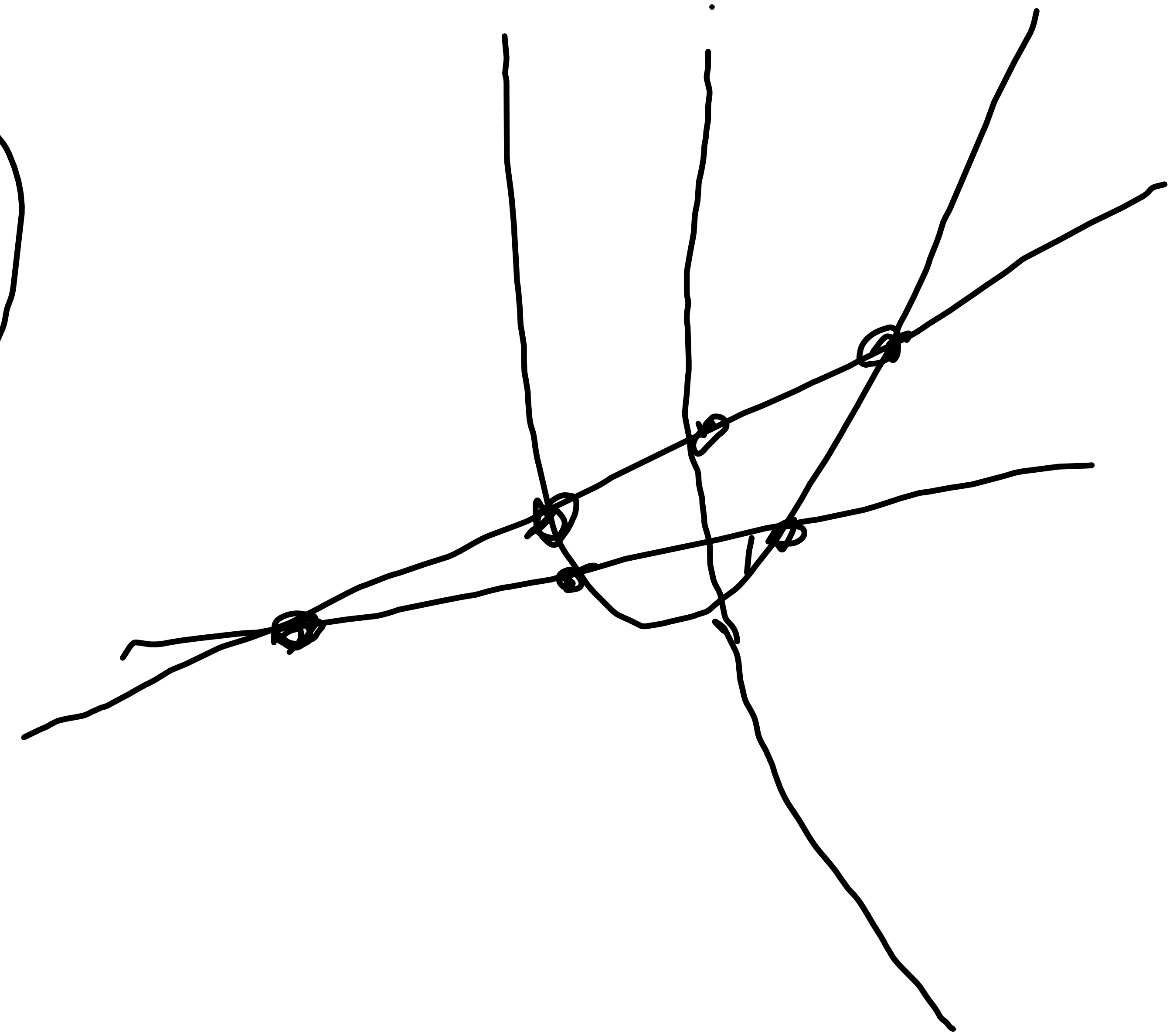
$y = x^3$

$$\begin{vmatrix} f'(u_0) & \varphi'(u_0) \\ f''(u_0) & \varphi''(u_0) \end{vmatrix} = 0$$

$\neq (0,0)$

$$x^2 + y^2 + ax + by + c = 0$$





$$l \geq 1$$

$$\begin{cases} l(l-2m)+m(m-2l)=0 \\ 9ln+n(2m-l)=0 \end{cases}$$

$$\begin{cases} l^2-2lm+m^2-2lm=0 \\ n(9l+2m-l)=0 \end{cases} \qquad \begin{cases} l^2-4lm+m^2=0 \\ n(8l+2m)=0 \end{cases}$$

$n=0$ (1) $\qquad m=-4l$ (2)

(1) $\begin{cases} l=2m\pm\sqrt{4m^2-m^2} = l=2m\pm m\sqrt{3} \\ n=0 \end{cases}$

$P_{1\infty}\equiv(0,2+\sqrt{3},1,0)$

$P_{2\infty}\equiv(0,2-\sqrt{3},1,0)$

(2) $\begin{cases} l^2+16l^2+16l^2=0 \\ m=-4l \end{cases} \quad \begin{cases} l=0 \\ m=0 \end{cases}$

$P_{3\infty}(0,0,0,1)$