Eg 11 $\quad C: \; 4x^2y - y^3 + 6xy - 8y + 1 = 0 \quad \cdot X_0^3$

Trovare asintoti

$F(X_0, X_1, X_2) = 4X_1^2X_2 - X_2^3 + 6X_1X_2\cancel{X_0} - 8X_2\cancel{X_0^2} + \cancel{X_0^3} = 0$

$F_0 = 6X_1X_2 - 16X_2X_0 + 3X_0^2$

$F_1 = 8X_1X_2 + 6X_2X_0$

$F_2 = 4X_1^2 - 3X_2^2 + 6X_1X_0 - 8X_0^2$

$\begin{cases} X_0 = 0 \\ 4X_1^2X_2 - X_2^3 = 0 \end{cases}$

$\begin{cases} X_2(4X_1^2 - X_2^2) = 0 \\ X_0 = 0 \end{cases}$

$\begin{cases} X_2(2X_1 + X_2)(2X_1 - X_2) = 0 \\ X_0 = 0 \end{cases}$

$A_\infty = (0,1,0) \begin{cases} X_2 = 0 \\ X_0 = 0 \end{cases}$

$B_\infty = (0,1,-2) \begin{cases} 2X_1 + X_2 = 0 \\ X_0 = 0 \end{cases}$

$C_\infty = (0,1,2) \begin{cases} 2X_1 - X_2 = 0 \\ X_0 = 0 \end{cases}$

	F_0	F_1	F_2
A_∞	0	0	4
B_∞	-12	-16	-8
C_∞	12	16	-8

Tang. in $(\bar{X}_0, \bar{X}_1, \bar{X}_2)$:

$X_0F_0 + X_1F_1 + X_2F_2 = 0$

F_0, F_1, F_2 calcolate in $(\bar{X}_0, \bar{X}_1, \bar{X}_2)$

in A_∞: $4X_2 = 0$

$4y = 0$

in B_∞:

$-12X_0 - 16X_1 - 8X_2 = 0$

$-12 - 16x - 8y = 0$

in C_∞:

$12X_0 + 16X_1 - 8X_2 = 0$

$12 + 16x - 8y = 0$

Es 12 $\quad \Sigma : x^4 - y^2 - z^2 = 0 \quad$ Trovare il piano

Π tang. a Σ in $A = (3, 9, 0)$

$F_x = 4x^3 \quad$ in A: 108

$F_y = -2y \quad -18$

$F_z = -2z \quad 0$

$\Pi : 108(x-3) - 18(y-9) + 0(z-0) = 0$

normale a Σ in A:

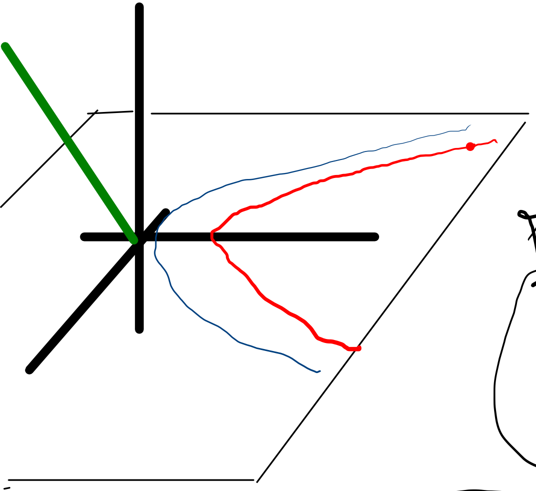
$$\cancel{\frac{x-3}{108} = \frac{y-9}{-18} = \frac{z-0}{0}} = \alpha$$

$$\begin{cases} z = 0 \\ \dfrac{x-3}{108} + \dfrac{y-9}{18} = 0 \end{cases}$$

$$\begin{cases} x = 108\alpha + 3 \\ y = -18\alpha + 9 \\ z = 0 \end{cases}$$

Es 10 $\mathcal{C}: \begin{cases} z=0 \\ y=x^2 \end{cases}$ Trovare le spf di rotazione di $\mathcal{C}$ attorno a: asse x, asse y, asse z e $r: \begin{cases} x=z \\ y=0 \end{cases}$

+1



Attorno all'asse y $\quad f(x,y)=x^2-y$

$$f\left(\pm\sqrt{x^2+z^2},\,y\right)=\left(\pm\sqrt{x^2+z^2}\right)^2-y$$

$$x^2+z^2-y=0$$

Attorno all'asse x

$$f\left(x,\pm\sqrt{y^2+z^2}\right)=x^2-\left(\pm\sqrt{y^2+z^2}\right)$$

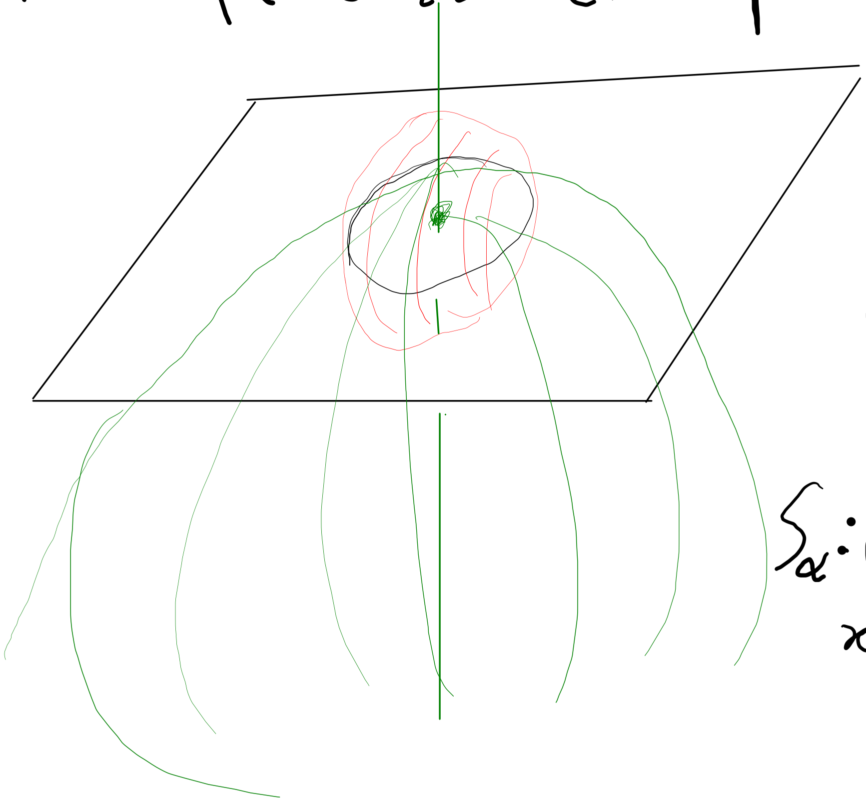
$$x^2=\pm\sqrt{y^2+z^2}$$

$$x^4=y^2+z^2 \qquad x^4-y^2-z^2=0$$

$P_\alpha \equiv (\alpha, \alpha^2, 0)$ $\quad r: \begin{cases} x = z \\ y = 0 \end{cases}$ $\quad (l, m, n) \sim (1, 0, 1)$

Piano π_α per P_α, $\perp r$ $\quad \pi_\alpha: 1(x-\alpha)+0(y-\alpha^2)+1(z-0)=0$

Circonferenza nello spazio: intersezione fra sfera e piano



Sfera S_α: passante per P_α e avente un centro comodo su r

Scelgo S_α per P_α con centro $O \equiv (0,0,0)$

$$S_\alpha: (x-0)^2+(y-0)^2+(z-0)^2=(\alpha-0)^2+(\alpha^2-0)^2+(0-0)^2$$

$$x^2+y^2+z^2=\alpha^2+\alpha^4$$

$$\Sigma: \begin{cases} x + z - \alpha = 0 \\ x^2 + y^2 + z^2 - \alpha^2 - \alpha^4 = 0 \end{cases} \qquad \alpha = (x + z)$$

$$\Sigma: x^2 + y^2 + z^2 - (x+z)^2 - (x+z)^4 = 0$$

Attorno all'asse z

$P_\alpha \equiv (\alpha, \alpha^2, 0)$ $\qquad S_\alpha : x^2 + y^2 + z^2 - \alpha^2 - \alpha^4 = 0$

Piano π_α per P_α, $\perp$ asse z : $0(x-\alpha) + 0(y-\alpha^2) + 1(z-0) = 0$

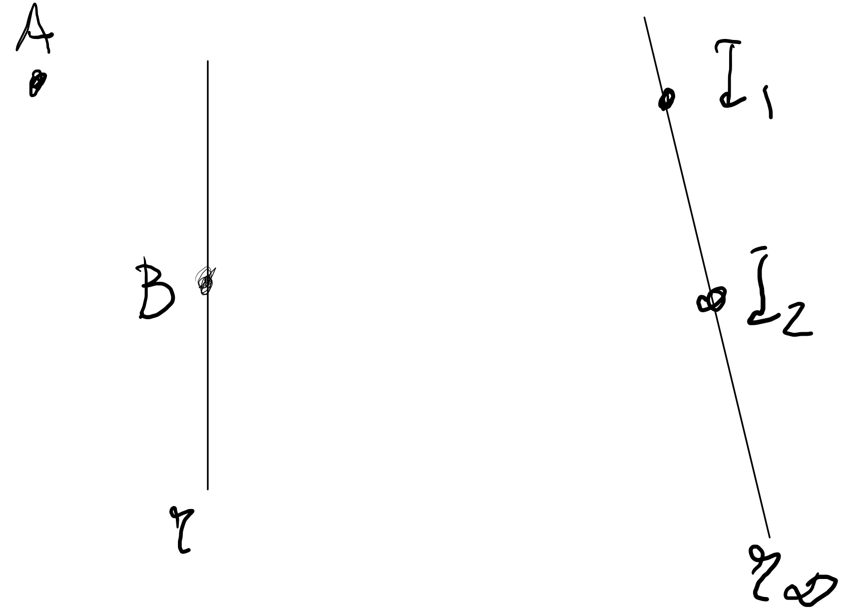
$\begin{cases} x = 0 \\ y = 0 \end{cases}$ $\quad (l, m, n) \sim (0,0,1)$ $\qquad z = 0$

$$\Sigma: \begin{cases} x^2 + y^2 + z^2 - \alpha^2 - \alpha^4 = 0 \\ z = 0 \end{cases} \longrightarrow z = 0$$

Es 10 $\quad \mathcal{C}$: circonf. ~~per~~ $A \equiv (2,0,0)$, tang. a $r : \begin{cases} z = -1 \\ y = 0 \end{cases}$ in $B \equiv (1,0,-1)$

Σ ottenuta ruotando $\mathcal{C}$ attorno all'asse z.

$\mathcal{C}$ giace sul piano $y=0$ (piano xz)



$\Gamma_1 = BI_1 \cup BI_2 \qquad BI_2 : z - ix + i - 1 = 0$

$\Gamma_2 = r \cup r_\infty$

$BI_1 : \dfrac{x-1}{1} = \dfrac{z+1}{i} \qquad ix - i = z + 1$

$z + ix - i + 1 = 0$

$\Gamma_1 : \big((z+1) + (ix - i)\big)\big((z+1) - (ix - i)\big) = 0$

$(z+1)^2 - (ix - i)^2 = 0$

$z^2 + 2z + 1 - (-x^2 + 2x - 1) = 0$

$\Gamma_1 : z^2 + x^2 + 2z - 2x + 2 = 0 \qquad \Gamma_2 : (X_3 + X_0) X_0$

$r \qquad r_\infty$

$\mathcal{F} : z^2 + x^2 + 2z - 2x + 2 + k(z+1) = 0 \qquad A = (2, 0, 0)$

$0 \quad 4 \quad 0 \quad -4 \quad +2 + k(0+1) = 0$

$2 + k = 0 \qquad k = -2$

$z^2 + x^2 + \cancel{2z} - 2x + \cancel{2} - \cancel{2z} - \cancel{2} = 0$

$C : \begin{cases} x^2 + z^2 - 2x = 0 \\ y = 0 \end{cases}$

attorno all'asse z :

$\left(\pm\sqrt{x^2+y^2}\right)^2 + z^2 - 2\left(\pm\sqrt{x^2+y^2}\right) = 0$

$x^2 + y^2 + z^2 = 2\left(\pm\sqrt{x^2+y^2}\right)$

$\Sigma : (x^2+y^2+z^2)^2 = 4(x^2+y^2)$