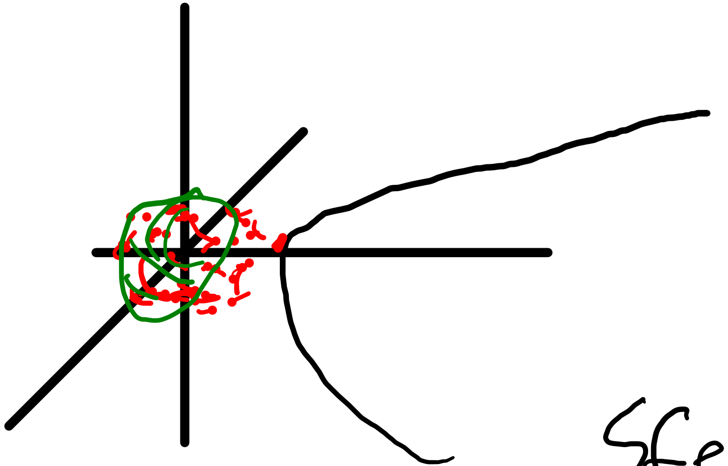


$$C: \begin{cases} z = 0 \\ y = x^2 + 1 \end{cases} \qquad P_\alpha \equiv (\alpha, \alpha^2+1, 0)$$

π_α per P_α, $\perp$ asse z

π_α = piano xy $(z=0)$ $\forall \alpha$

Sfera S_α per P_α, centro in $O \equiv (0,0,0)$

$$S_\alpha : (x-0)^2 + (y-0)^2 + (z-0)^2 = (\alpha-0)^2 + (\alpha^2+1-0)^2 + (0-0)^2$$

$$x^2+y^2+z^2 = \alpha^2 + \alpha^4 + 2\alpha^2 + 1$$

$$\alpha^4 + 3\alpha^2 + 1 \longrightarrow z = 0$$

$$\begin{matrix} \pi_\alpha: \\ S_\alpha: \end{matrix} \begin{cases} z = 0 \\ x^2+y^2+z^2-\alpha^4-3\alpha^2-1 = 0 \end{cases}$$

$\alpha = \frac{i}{2}$ $\quad \alpha^2 = -\frac{1}{4}$

$\alpha^4 + 3\alpha^2 + 1 =$

$= \frac{1}{16} - \frac{3}{4} + 1 =$

$= \frac{1-12+16}{16} = \frac{5}{16}$

$R_0 = \frac{\sqrt{5}}{4} < 1$

si attraversano

$$y = f(x) = a_1 x + a_2 x^2 + \dots + a_n x^n + a_{n+1} x^{n+1} + \dots$$

$$y = \varphi(x) = b_1 x + b_2 x^2 + \dots + b_n x^n + b_{n+1} x^{n+1} + \dots$$

contatto di ordine n

$$a_1 = b_1, \dots, a_n = b_n \quad a_{n+1} \neq b_{n+1}$$

$$\varphi(x) - f(x) = 0x + 0x^2 + \dots 0x^n + (b_{n+1} - a_{n+1}) x^{n+1} + \dots$$

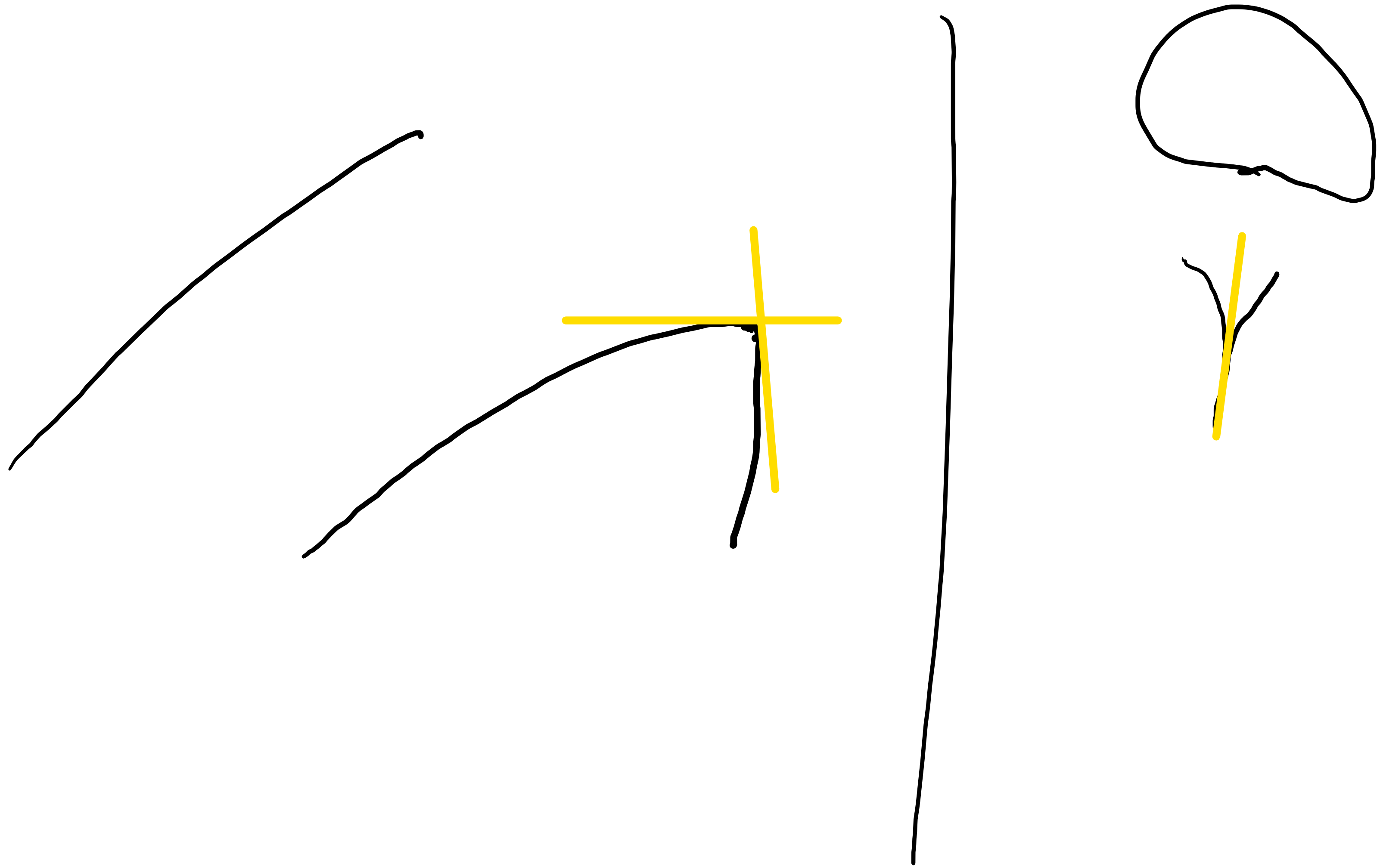
$x \to 0$

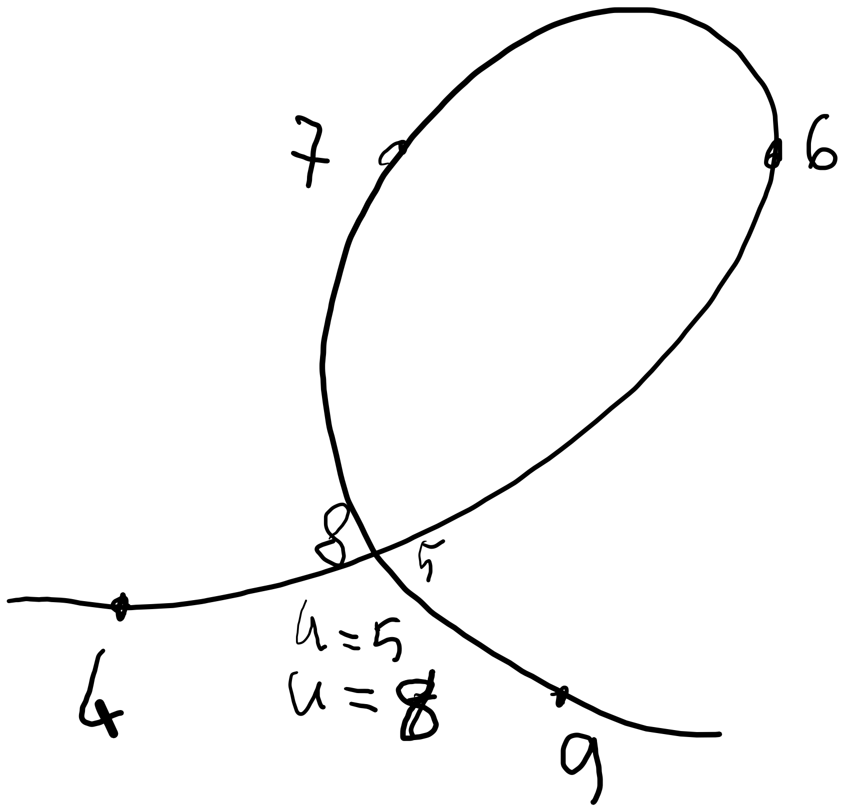
in un opportuno intorno di 0, questo differenza è approssimabile da

$$(b_{n+1} - a_{n+1}) x^{n+1}$$

questa differenza assume segni diversi a sinistra e a destra di 0 $\Leftrightarrow$ n+1 è dispari $\Leftrightarrow$ n è pari.

segni uguali $\Leftrightarrow$ n+1 è pari $\Leftrightarrow$ n è dispari




7
6
8
5
4
u=5
u=8
9

Es 9 $\mathcal{C}: y = 1/x$.

$$\begin{cases} x = u \\ y = \dfrac{1}{u} \end{cases}$$

a) circ. osculatrici in $P \equiv (2, 1/2)$, $Q \equiv (1, 1)$.

b) ordine di contatto

$$\begin{cases} x = u \\ y = 1/u \end{cases}$$

$$x^2 + y^2 + ax + by + c = 0 \quad f(x,y) =$$

$$\Phi(u) = f(u, 1/u) = u^2 + \frac{1}{u^2} + au + b\frac{1}{u} + c$$

$$\Phi'(u) = 2u - \frac{b}{u^2} - \frac{3}{u^3} + a$$

$$\Phi''(u) = \frac{2b}{u^3} + \frac{6}{u^4} + 2$$

$$\Phi'''(u) = -\frac{6b}{u^4} - \frac{24}{u^5}$$

$$\Phi^{IV}(u) = \frac{24b}{u^5} + \frac{120}{u^6}$$

P	Q
$u = 2$	$u = 1$
$c + \frac{b}{2} + 2a + \frac{17}{4}$	$c + b + a + 2$
$-\frac{b}{4} + a + \frac{15}{4}$	$a - b$
$\frac{b}{4} + \frac{19}{8}$	$2b + 8$
$-\frac{3b}{8} - \frac{3}{4}$	$6b - 24$
$\frac{3}{4}b + \frac{15}{8}$	$24b + 120$

	P	Q
	$u=2$	$u=1$
C	$c+\frac{b}{2}+2a+\frac{13}{4}$	$c+b+a+2$
	$-\frac{b}{4}+a+\frac{15}{4}$	$a-b$
	$\frac{b}{4}+\frac{19}{8}$	$2b+8$
	$-\frac{3b}{8}-\frac{3}{4}$	$6b-24$
	$\frac{3}{4}b+\frac{15}{8}$	$24b+120$

$$\begin{cases}\Phi(\bar{u})=0\\ \Phi'(\bar{u})=0\\ \Phi''(\bar{u})=0\end{cases}$$

P
$$\begin{cases}2a+\frac{b}{2}+c+\frac{17}{4}=0\\ a-\frac{b}{4}+\frac{15}{4}=0\\ \frac{b}{4}+\frac{19}{8}=0\end{cases}$$

$$\begin{cases}a=-\frac{49}{8}\\ b=-\frac{19}{2}\\ c=\frac{51}{4}\end{cases}$$

$$\Phi'''(2)=\frac{45}{16}\neq 0$$

c. osc.:

$$x^2+y^2-\frac{49}{8}x-\frac{19}{2}y+\frac{51}{4}=0$$

ordine di cont. = 2

Q
$$\begin{cases}a+b+c+2=0\\ a-b=0\\ 2b+8=0\end{cases}$$

$$\begin{cases}c=6\\ a=-4\\ b=-4\end{cases}$$

$$\Phi'''(1)=0$$

$$\Phi^{IV}(1)=-96+120\neq 0$$

c. osc.:

$$x^2+y^2-4x-4y+6=0$$

ordine di cont. = 3

$$(x-x_0)^2+(y-y_0)^2-R^2=0$$

$$x^2-2x_0x+x_0^2+y^2-2y_0y+y_0^2-R^2=0$$

$$x^2+y^2\underbrace{-2x_0}_{a}x\underbrace{-2y_0}_{b}y+\underbrace{(x_0^2+y_0^2-R^2)}_{c}=0$$

$$x_0=-\frac{a}{2}$$

$$y_0=-\frac{b}{2}$$

$$\left(-\frac{a}{2}\right)^2+\left(-\frac{b}{2}\right)^2-R^2=c$$

$$R^2=\frac{a^2}{4}+\frac{b^2}{4}-c$$

$$x_0=-\frac{-4}{2}=2$$

$$y_0=\frac{2}{2}$$

$$R^2=\frac{16}{4}+\frac{16}{4}-6=2$$

$$R=\sqrt{2}$$

$$\text{curvatura}=\frac{1}{\sqrt{2}}$$