5 c

$C : \begin{cases} x = u^3 - 6u^2 \\ y = 3u \\ z = u - 1 \end{cases}$ Luogo $\mathcal{L}$ delle tang. a C

$x' = 3u^2 - 12u$
$y' = 3$
$z' = 1$

$$t_u : \frac{x - u^3 + 6u^2}{3u^2 - 12u} = \frac{y - 3u}{3} = \frac{z - u + 1}{1} = v$$

$$\mathcal{L} : \begin{cases} x = u^3 - 6u^2 + v(3u^2 - 12u) \\ y = 3u + 3v \\ z = u - 1 + v \end{cases} \quad \begin{cases} - \\ - \\ v = z - u + 1 \end{cases}$$

$$\begin{cases} x = u^3 - 6u^2 + (3u^2 - 12u)(z - u + 1) \\ y = 3u + 3(z - u + 1) = 3z + 3 \end{cases} \quad \cancel{\ } \rightsquigarrow y = 3z + 3$$

$\pi : \begin{cases} y = 3 \\ z = 9x \end{cases}$ $\begin{cases} x = \alpha \\ y = 3 \\ z = 9\alpha \end{cases}$ $(l, m, n) \sim (1, 0, 9)$

impongo $1 \cdot (3u^2 - 12u) + 0 \cdot 3 + 9 \cdot 1 = 0$

11/5/09 3 a b

$C: \begin{cases} x = \alpha \\ y = \sin\alpha \\ z = \alpha \end{cases}$

a) piano osc. in ogni punto C

b) verificare se C è piana, trovare il piano

$P_\alpha \equiv (\alpha, \sin\alpha, \alpha)$

$x' = 1 \qquad x'' = 0$

$y' = \cos\alpha \qquad y'' = -\sin\alpha$

$z' = 1 \qquad z'' = 0$

$$\Pi_\alpha : \begin{vmatrix} (x-\alpha) & (y-\sin\alpha) & (z-\alpha) \\ 1 & \cos\alpha & 1 \\ 0 & -\sin\alpha & 0 \end{vmatrix} = 0$$

$$\Pi_\alpha : \sin\alpha \begin{vmatrix} (x-\alpha) & (z-\alpha) \\ 1 & 1 \end{vmatrix} = 0$$

$\sin\alpha(x - \alpha - z + \alpha) = 0$

$\sin\alpha(x - z) = 0$

$\sin\alpha \neq 0 \longrightarrow x - z = 0$

20/3/09 2b

$$C: \begin{cases} x = \alpha \\ y = \alpha^4 - \alpha^2 \\ z = \alpha^2 \end{cases}$$

Triedro princ. in $A \equiv (1, 0, 1)$

$\alpha = 1$

$x' = 1$		1
$y' = 4\alpha^3 - 2\alpha$		2
$z' = 2\alpha$		2
$x'' = 0$		0
$y'' = 12\alpha^2 - 2$		10
$z'' = 2$		2

$$t: \frac{x-1}{1} = \frac{y-0}{2} = \frac{z-1}{2} \qquad \alpha = 1$$

$$\pi_o: \begin{vmatrix} (x-1) & y & (z-1) \\ 1 & 2 & 2 \\ 0 & 10 & 2 \end{vmatrix} = 0$$

$$-16(x-1) - 2y + 10(z-1) = 0$$

$$-16x - 2y + 10z + 6 = 0$$

$$8x + y - 5z - 3 = 0$$

$$\pi_n: 1(x-1) + 2(y) + 2(z-1) = 0$$

$$x + 2y + 2z - 3 = 0$$

$$n: \begin{cases} 8x + y - 5z - 3 = 0 \\ x + 2y + 2z - 3 = 0 \end{cases} \qquad \begin{pmatrix} 8 & 1 & -5 \\ 1 & 2 & 2 \end{pmatrix}$$

$$(l, m, n) \sim \left(\begin{vmatrix} 1 & -5 \\ 2 & 2 \end{vmatrix}, -\begin{vmatrix} 8 & -5 \\ 1 & 2 \end{vmatrix}, \begin{vmatrix} 8 & 1 \\ 1 & 2 \end{vmatrix} \right) = (12, -21, 15) \sim (4, -7, 5)$$

$$\pi_r: 4(x-1) - 7(y) + 5(z-1) = 0$$

$$4x - 7y + 5z - 9 = 0$$

$$b: \begin{cases} x + 2y + 2z - 3 = 0 \\ 4x - 7y + 5z - 9 = 0 \end{cases}$$

Formule di Frenet $\vec{t}$ versore della tang.

$\varkappa$ flessione $\vec{n}$ " " norm. pr.

τ torsione $\vec{b}$ " " binormale

$$\begin{pmatrix} \vec{t}\,' \\ \vec{n}\,' \\ \vec{b}\,' \end{pmatrix} = \begin{pmatrix} 0 & \varkappa & 0 \\ -\varkappa & 0 & -\tau \\ 0 & \tau & 0 \end{pmatrix} \begin{pmatrix} \vec{t} \\ \vec{n} \\ \vec{b} \end{pmatrix}$$

Circ. osculatrice a $\mathcal{C}$: $\begin{cases} x = \alpha \\ y = \alpha^2 \\ z = \alpha^3 \end{cases}$ in $O \equiv (0,0,0)$ ($\alpha = 0$)

in O

$x' = 1$ 1

$y' = 2\alpha$ 0

$z' = 3\alpha^2$ 0

$\pi_O : \begin{vmatrix} x & y & z \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 0 \quad 2z = 0 \quad z = 0$

$x'' = 0$ 0

$y'' = 2$ 2

$z'' = 6\alpha$ 0

Generica sfera: $\underbrace{x^2 + y^2 + z^2 + ax + by + cz + d}_{F(x,y,z)} = 0$

$\Phi(\alpha) = \alpha^2 + \alpha^4 + \alpha^6 + a\alpha + b\alpha^2 + c\alpha^3 + d$ $\Phi(0) = d$

$\Phi'(\alpha) = 2\alpha + 4\alpha^3 + 6\alpha^5 + a + 2b\alpha + 3c\alpha^2$ $\Phi'(0) = a$

$\Phi''(\alpha) = 2 + 12\alpha^2 + 30\alpha^4 + 2b + 6c\alpha$ $\Phi''(0) = 2 + 2b$

$\begin{cases} d = 0 \\ a = 0 \\ b = -1 \end{cases}$

Generica sfera S_c: $x^2 + y^2 + z^2 - y + cz = 0$ ← c variabile

Un'espressione della circ. osc.

$\begin{cases} z = 0 \\ x^2 + y^2 + z^2 - y + cz = 0 \end{cases}$

centro di S_c: $P_c \equiv \left(0, \frac{1}{2}, -\frac{c}{2}\right)$

raggio di S_c: distanza centro punto

$R_c = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{c}{2}\right)^2}$

Se voglio il raggio di curvatura cerco la sfera osculatrice con centro sul piano osculatore

Impongo a P_c di stare su $z = 0$: $-\frac{c}{2} = 0$, $c = 0$

So $R_0 = \sqrt{\frac{1}{4}} = \frac{1}{2}$ $\kappa = \frac{1}{1/2} = 2$

Es 22 $S: \underbrace{x^3-2y^2-2z-4x+4}_{f(x,y,z)}=0$ Tang. asint. in $P\equiv(2,0,2)$

Generica retta per P, $\begin{cases} x=2+\ell u \\ y=mu \\ z=2+nu \end{cases}$ $P \Leftrightarrow u=0$

$\Phi(u)=(2+\ell u)^3-2(mu)^2-2(2+nu)-4(2+\ell u)+4=$

$=\ell^3u^3-2m^2u^2+6\ell^2u^2-2nu+8\ell u$

per $u=0$

0

$\Phi'(u)=3\ell^3u^2-4m^2u+12\ell^2u-2n+8\ell$

$-2n+8\ell$

$\Phi''(u)=6\ell^3u-4m^2+12\ell^2$

$-4m^2+12\ell^2$

$\begin{cases} n=4\ell \\ m^2=3\ell^2 \end{cases}$ $\begin{cases} n=4\ell \\ m=\pm\sqrt{3}\,\ell \end{cases}$

scelgo $\ell=1$

Tang. as. in P:

$\begin{cases} x=2+u \\ y=\pm\sqrt{3}\,u \\ z=2+4u \end{cases}$

Es 14 Σ $F = x^4+y^4+z^4+2x^2z^2+2y^2z^2-14x^2-14y^2-10z^2+25 = 0$

$$\begin{cases} F=0 \\ F_x=0 \\ F_y=0 \\ F_z=0 \end{cases}$$

$F_x = 4x^3+4xz^2-28x = 4x(x^2+z^2-7)$

$F_y = 4y^3+4yz^2-28y = 4y(y^2+z^2-7)$

$F_z = 4z^3+4zy^2-20z+4zx^2 = 4z(z^2+y^2+x^2-5)$

$(0,0,\pm\sqrt{5})$

$$\begin{cases} F=0 \\ x(x^2+z^2-7)=0 \\ y(y^2+z^2-7)=0 \\ z(x^2+y^2+z^2-5)=0 \end{cases}$$

$$\begin{cases} F=0 \\ x=0 \\ y=0 \\ z=0 \end{cases} \quad \begin{cases} 25=0 \\ = \\ = \end{cases}$$

$$\begin{cases} F=0 \\ x=0 \\ y=0 \\ x^2+y^2+z^2-5=0 \end{cases} \quad \begin{cases} F=0 \\ x=0 \\ y=0 \\ z=\pm\sqrt{5} \end{cases}$$

$$\begin{cases} F=0 \\ x^2+z^2-7=0 \\ y=0 \\ z=0 \end{cases} \quad \begin{cases} F=0 \\ x=\pm\sqrt{7} \\ y=0 \\ z=0 \end{cases} \quad \begin{cases} -24 \\ = \\ = \end{cases}$$

$$\begin{cases} F=0 \\ x=0 \\ y^2+z^2-7=0 \\ x^2+y^2+z^2-5=0 \end{cases}$$

$$\begin{cases} F=0 \\ x=0 \\ y^2+z^2-7=0 \\ z=0 \end{cases} \quad \begin{cases} F=0 \\ x=0 \\ y=\pm\sqrt{7} \\ z=0 \end{cases} \quad \begin{cases} -24=0 \\ = \\ = \\ = \end{cases}$$

$$\begin{cases} F=0 \\ x^2+z^2-7=0 \\ y=0 \\ x^2+y^2+z^2-5=0 \end{cases}$$

$$\begin{cases} F=0 \\ x^2+z^2-7=0 \\ y^2+z^2-7=0 \\ z=0 \end{cases} \quad \begin{cases} F=0 \\ x^2=7 \\ y^2=7 \\ z=0 \end{cases} \quad \begin{cases} -73=0 \end{cases}$$

$$\begin{cases} F=0 \\ x^2+z^2-7=0 \\ y^2+z^2-7=0 \\ x^2+y^2+z^2-5=0 \end{cases} \quad \begin{cases} F=0 \\ x^2=7-z^2 \\ y^2=7-z^2 \end{cases}$$

$$\begin{cases} F=0 \\ = \\ 7-z^2+7-z^2+z^2-5=0 \end{cases} \quad \begin{cases} x^2=7-z^2 \\ y^2=7-z^2 \\ 9-z^2=0 \end{cases} \quad \begin{cases} x^2=-2 \\ y^2=-2 \\ z^2=9 \end{cases}$$

$F_x = 4x^3 + 4xz^2 - 28x = 4x(x^2 + z^2 - 7)$

$F_y = 4y^3 + 4yz^2 - 28y = 4y(y^2 + z^2 - 7)$

$F_z = 4z^3 + 4zy^2 - 20z + 4zx^2 = 4z(z^2 + y^2 + x^2 - 5)$

$\nabla F = 0$ $\quad (0, 0, \pm\sqrt{5})$ $\quad (0, 0, \sqrt{5})$

$F_{xx} = 12x^2 + 4z^2 - 28$ $\quad -8$

$F_{xy} = 0$ $\quad 0$

$F_{xz} = 8xz$ $\quad 0$

$F_{yy} = 12y^2 + 4z^2 - 28$ $\quad -8$

$F_{yz} = 8yz$ $\quad 0$

$F_{zz} = 12z^2 + 4y^2 - 20 + 4x^2$ $\quad 40$

$-8(x-0)^2 + 2\cdot 0(x-0)(y-0) + 2\cdot 0(x-0)(z-\sqrt{5}) - 8(y-0)^2 + 2(y-0)(z-\sqrt{5}) + 40(z-\sqrt{5})^2 = 0$

$-8x^2 - 8y^2 + 40(z-\sqrt{5})^2 = 0$