In uno sp. affine $\mathcal{A}^3$

1) Data la retta r: $\begin{cases} x-2y+z-3=0 \\ x+3y-2z+1=0 \end{cases}$, data il punto $P\equiv(0,-1,0)$, trovare il piano che contiene r e P.

Trovo il fascio di piani contenenti r:

$$\alpha(x-2y+z-3)+\beta(x+3y-2z+1)=0$$

$$(\alpha+\beta)x+(-2\alpha+3\beta)y+(\alpha-2\beta)z-3\alpha+\beta=0$$

Impongo passaggio per P:

$$\overset{0}{} \quad \overset{-1}{} \quad \overset{0}{}$$

$$2\alpha-3\beta \quad -3\alpha+\beta=0$$

$$-\alpha-2\beta=0$$

$$(\alpha,\beta)\sim(2,-1)$$

$$x-7y+4z-7=0$$

2) r la stessa retta ; $s: \begin{cases} x = y-1 \\ y = 5 \\ z = 2y+1 \end{cases}$ $(l,m,n) \sim (1,0,2)$

Trovare il piano contenente r e parallelo ad s

Fascio di piani contenenti r:

$$\underbrace{(\alpha+\beta)}_{a=}x + \underbrace{(-2\alpha+3\beta)}_{b=}y + \underbrace{(\alpha-2\beta)}_{c=}z - 3\alpha+\beta = 0$$

il piano è parallelo ad $s \iff al + bm + cn = 0$

$\iff (\alpha+\beta)\cdot 1 + (-2\alpha+3\beta)\cdot 0 + (\alpha-2\beta)\cdot 2 = 0$

$\iff 3\alpha - 3\beta = 0 \iff \alpha = \beta$ $\quad (\alpha,\beta) \sim (1,1)$

$$2x + y - z - 2 = 0$$

Passo all'ampliamento proiettivo

1) $r: \begin{cases} -3X_0 + X_1 - 2X_2 + X_3 = 0 \\ X_0 + X_1 + 3X_2 - 2X_3 = 0 \end{cases}$ $P \equiv (1, 0, -1, 0)$

Fascio $\alpha(-3X_0 + X_1 - 2X_2 + X_3) + \beta(X_0 + X_1 + 3X_2 - 2X_3) = 0$

$$(-3\alpha+\beta)X_0 + (\alpha+\beta)X_1 + (-2\alpha+3\beta)X_2 + (\alpha-2\beta)X_3 = 0$$

$$\qquad\qquad 1 \qquad\qquad\quad 0 \qquad\qquad\quad -1 \qquad\qquad 0$$

Passaggio per P:

$$-\alpha - \beta = 0 \qquad (\alpha, \beta) \sim (2, -1)$$

$$-2X_0 + 2X_1 + X_2 - X_3 = 0$$

$$2x + y - z - 2 = 0$$

2) Punto improprio di s : $S_\infty \equiv (0,1,0,2)$

Fascio: $(-3\alpha+\beta)\underset{0}{X_0}+(\alpha+\beta)\underset{1}{X_1}+(-2\alpha+3\beta)\underset{0}{X_2}+(\alpha-2\beta)\underset{2}{X_3}=0$

Impongo il passaggio per S_∞ :

$$\alpha+\beta+2\alpha-4\beta=0$$

$$3\alpha-3\beta=0 \qquad \alpha=\beta$$

$$(\alpha,\beta)\sim(1,1)$$