$$X_0^2 - X_1^2 = 0 \qquad \sigma = (1,1)$$

$$(X_0 + X_1)(X_0 - X_1) = 0$$

$$X_0^2 = 0 \qquad \sigma = (1,0) \circ (0,1)$$

$$X_0^2 + X_1^2 - X_2^2 = 0$$

$$4x^2+9y^2+12xy+4x+6y+1=0$$

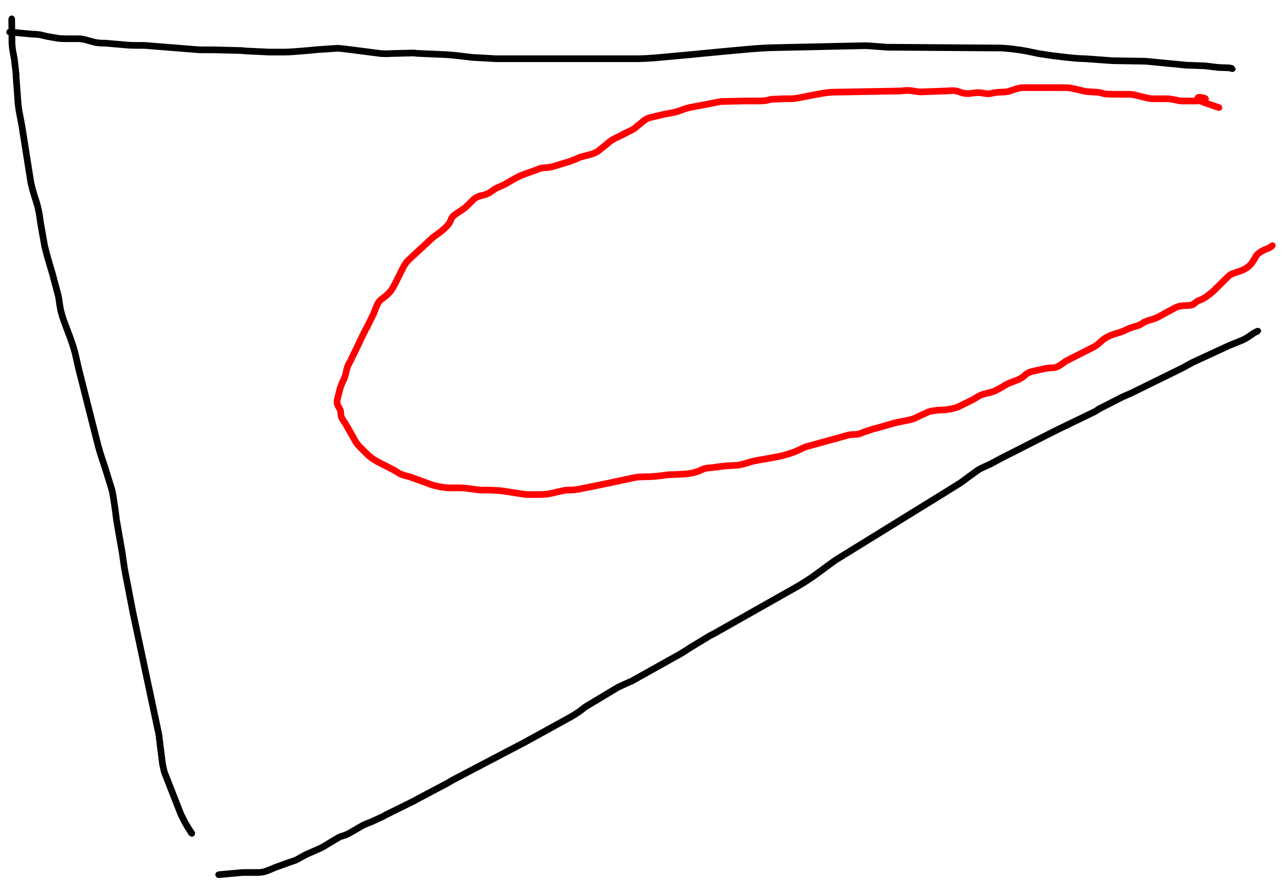
$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix} \quad \text{rango} = 1$$

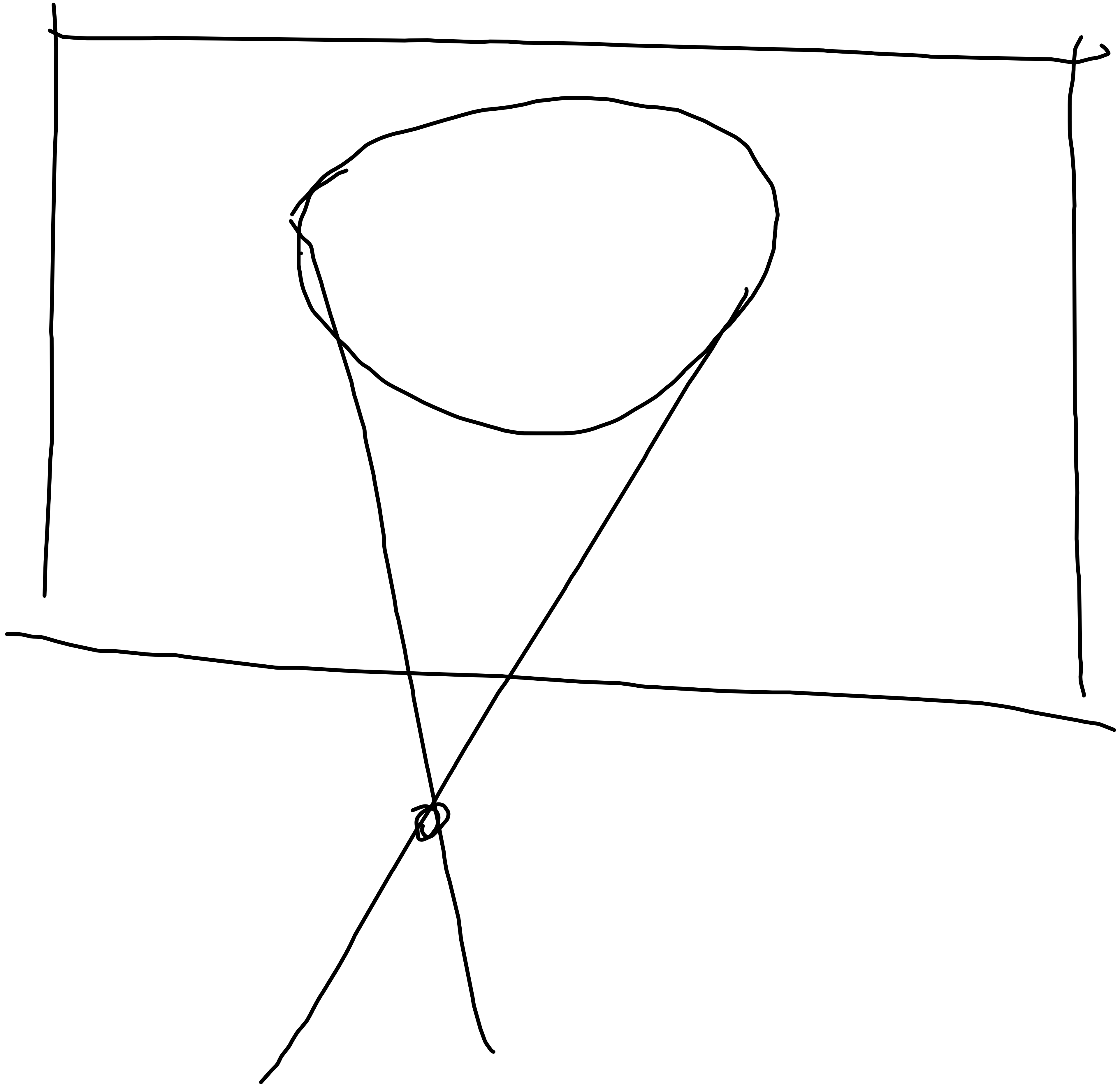
$$x^2-2xy-3y^2-2x+6y=0 \qquad A=\begin{pmatrix} 0 & -1 & 3 \\ -1 & 1 & -1 \\ 3 & -1 & -3 \end{pmatrix}$$

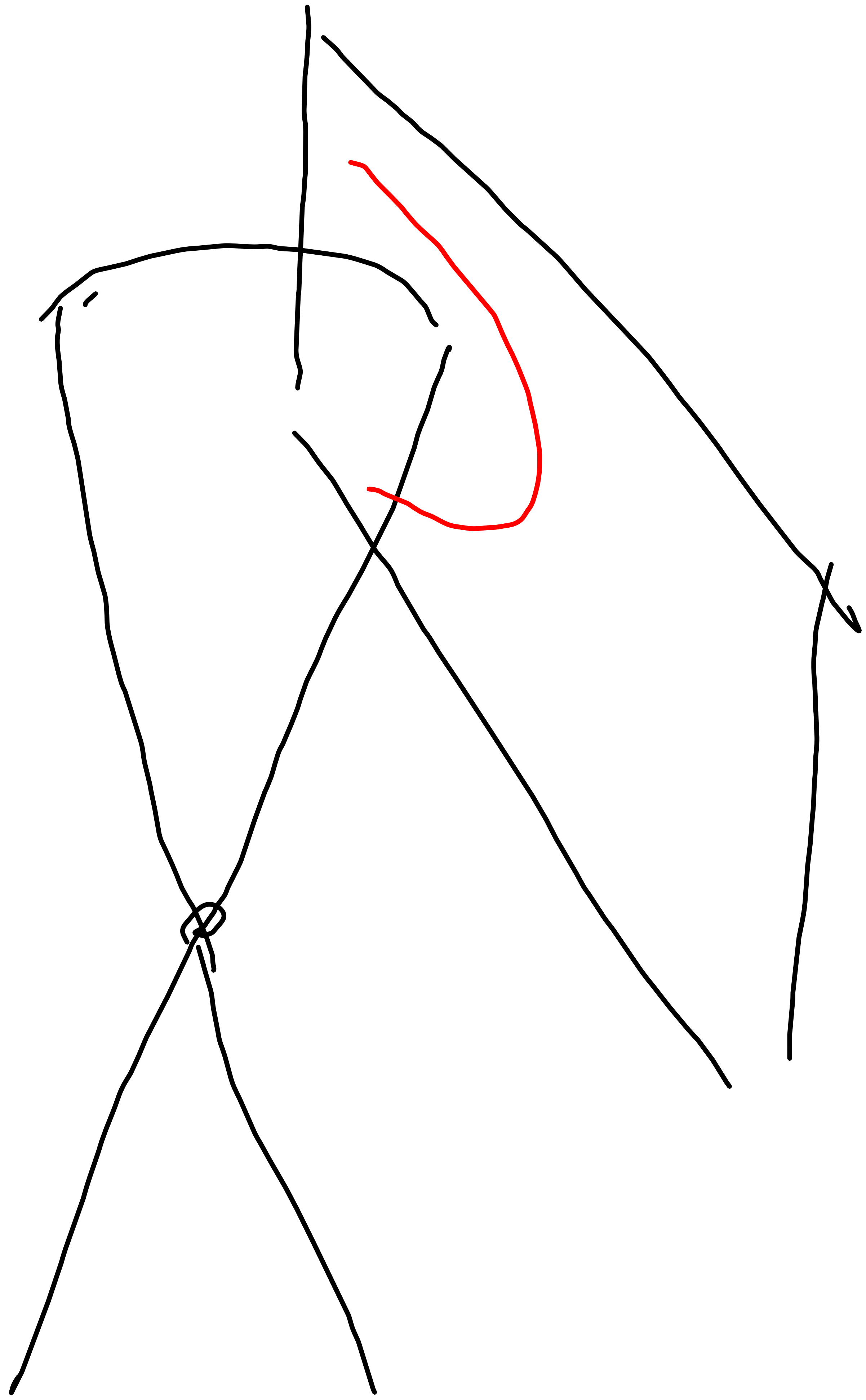
$|A|=0$ deg.

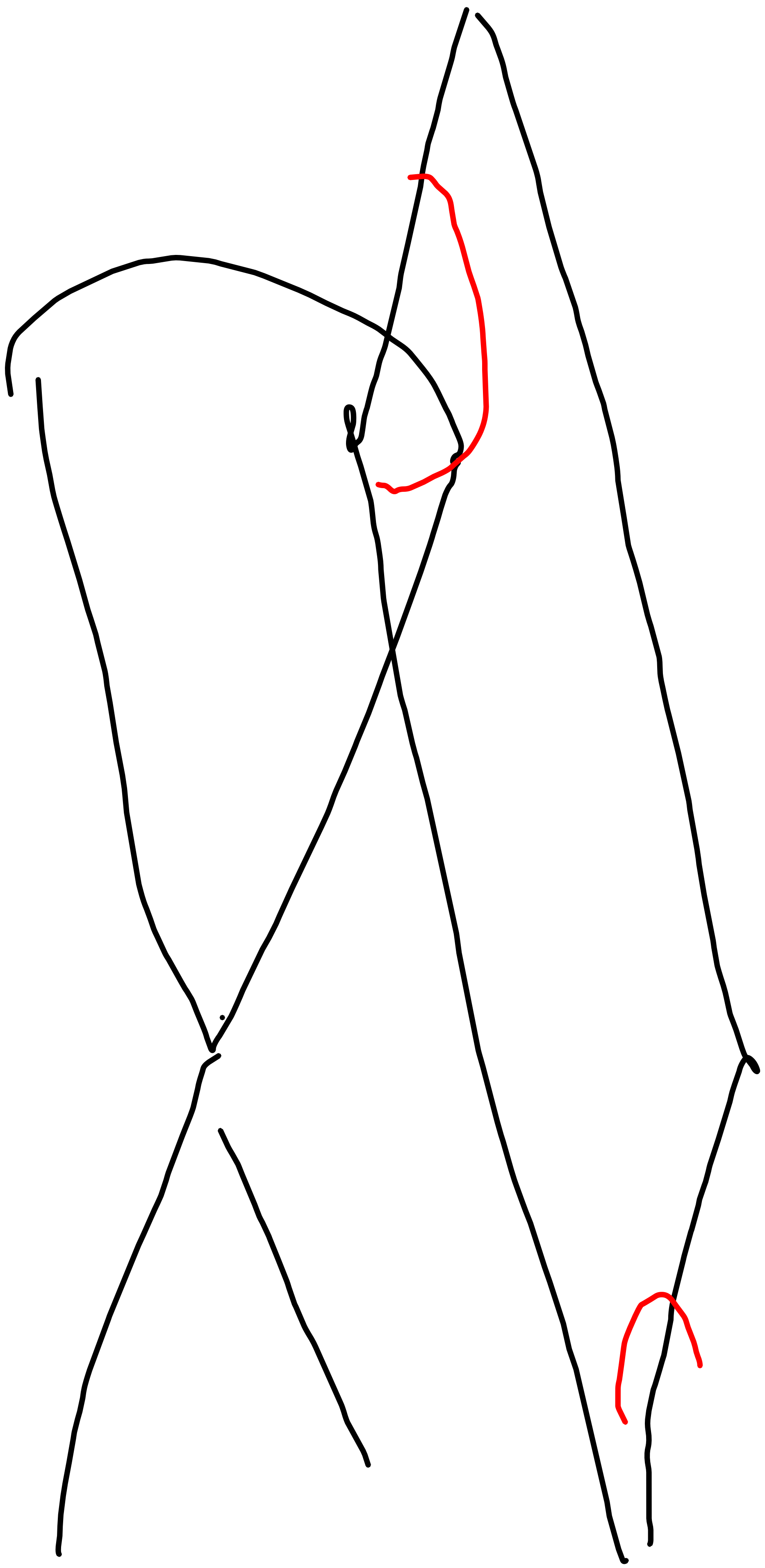
pol. car.

$$\lambda^3+2\lambda^2-14\lambda = \lambda(\lambda^2+2\lambda-14) \qquad \sigma=(1,1)$$









Affinità: trasformazione regolare della spazio affine del tipo:

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = A \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

$\det A \neq 0$

$n=2$

$$\begin{cases} y_1 = a_1^1 x_1 + a_2^1 x_2 + b_1 \\ y_2 = a_1^2 x_1 + a_2^2 x_2 + b_2 \end{cases}$$

$$Y_0 \left\{ \begin{aligned} y_1 &= a_1^1 x_1 + a_2^1 x_2 + b_1 \\ y_2 &= a_1^2 x_1 + a_2^2 x_2 + b_2 \end{aligned} \right. \quad \cdot X_0$$

$$y_1 = \frac{Y_1}{Y_0}$$

$$y_2 = \frac{Y_2}{Y_0}$$

$$x_1 = \frac{X_1}{X_0}$$

$$x_2 = \frac{X_2}{X_0}$$

$$\lambda \left\{ \begin{aligned} Y_0 &= X_0 \\ Y_1 &= b_1 X_0 + a_1^1 X_1 + a_2^1 X_2 \\ Y_2 &= b_2 X_0 + a_2^1 X_1 + a_2^2 X_2 \end{aligned} \right.$$

$$\lambda \begin{pmatrix} Y_0 \\ Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ b_1 & a_1^1 & a_2^1 \\ b_2 & a_1^2 & a_2^2 \end{pmatrix} \begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix}$$

Classificazione proiettiva delle iperquadriche di $\mathbb{P}^1(\mathbb{R})$

$\sigma = (1,0)$ o $(0,1)$ $\quad$ $\mathrm{Im}: X_0^2 = 0$ $\quad$ Un punto ("contato 2 volte")

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad$ $W: \begin{cases} X_0 = 0 \\ 0 = 0 \end{cases}$ $\quad$ lo stesso punto

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$\sigma = (2,0)$ o $(0,2)$ $\quad$ $\mathrm{Im}: X_0^2 + X_1^2 = 0$ $\quad$ $\emptyset$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad$ $W: \begin{cases} X_0 = 0 \\ X_1 = 0 \end{cases}$ $\emptyset$

$\sigma = (1,1)$ $\quad$ $\mathrm{Im}: X_0^2 - X_1^2 = 0$ $\quad$ Due punti

$(X_0 + X_1)(X_0 - X_1) = 0$

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad$ $W: \begin{cases} X_0 = 0 \\ -X_1 = 0 \end{cases}$ $\emptyset$

Conica generica:

$$\begin{cases} \cancel{a^0_0 X_0^2} + 2\cancel{a^0_1 X_0 X_1} + 2\cancel{a^0_2 X_0 X_2} + a^1_1 X_1^2 + 2a^1_2 X_1 X_2 + a^2_2 X_2^2 = 0 \\ X_0 = 0 \end{cases}$$

rappresenta l'iperquadrica $\cap$ con M_∞

retta impropria del piano

$$A = \begin{pmatrix} a^0_0 & a^0_1 & a^0_2 \\ a^1_0 & \boxed{\begin{matrix} a^1_1 & a^1_2 \\ a^2_1 & a^2_2 \end{matrix}} \\ a^2_0 & & \end{pmatrix}$$

$$a^1_2 = \begin{pmatrix} a^1_1 & a^1_2 \\ a^2_1 & a^2_2 \end{pmatrix} = M_\infty$$

$$A_{\infty} = A^0_0 = |M^0_0| = |M_\infty| = 0 \iff \sigma = (1,0) \text{ o } (0,1) \quad \text{1 punto contato 2 volte}$$

$$> 0 \iff \sigma = (2,0) \text{ o } (0,2) \quad \emptyset$$

$$< 0 \iff \sigma = (1,1) \quad \text{2 punti}$$

Classificare affinemente, al variare di $\gamma \in \mathbb{R}$, le coniche di eq.

$x^2 + 2\gamma xy + 4y^2 + 20y - 10 = 0$

$A = \begin{pmatrix} -10 & 0 & 10 \\ 0 & 1 & \gamma \\ 10 & \gamma & 4 \end{pmatrix}$ rango ≥ 2

$M^0_a =$

$|A| = 10(\gamma^2 - 14) = 0 \iff \gamma = \pm\sqrt{14}$

deg. di rango $= 2$

$M^0_a = \begin{pmatrix} 1 & \gamma \\ \gamma & 4 \end{pmatrix}$ $|M^0_a| = 4 - \gamma^2$ $>$ $-2 < \gamma < 2$

$= 0$ $\gamma = \pm 2$

$<$ $\gamma < -2 \vee \gamma > 2$

segni discordi sulla diag. princ. sempre INDEFINITA

γ	$\|A\|$	rango A	A_0^0	conica
$\gamma < -\sqrt{14}$	$\neq 0$	3	$-$	iperboli
$\gamma = -\sqrt{14}$	$= 0$	2	$-$	deg r=2
$-\sqrt{14} < \gamma < -2$	$\neq 0$	3	$-$	iperboli
$\gamma = -2$	$\neq 0$	3	0	parabola
$-2 < \gamma < 2$	$\neq 0$	3	$+$	ellissi reali
$\gamma = 2$	$\neq 0$	3	0	parabola
$2 < \gamma < \sqrt{14}$	$\neq 0$	3	$-$	iperboli
$\gamma = \sqrt{14}$	$= 0$	2	$-$	deg r=2
$\gamma > \sqrt{14}$	$\neq 0$	3	$-$	iperboli