$$\begin{cases} x = \alpha + 2\beta \\ y = \alpha^2 + 4\alpha\beta + 4\beta^2 + 1 \\ z = -\alpha - 2\beta + 2 \end{cases}$$

$$\begin{pmatrix} 1 & (2\alpha + 4\beta) & -1 \\ 2 & (4\alpha + 8\beta) & -2 \end{pmatrix} \begin{array}{l} \cdot 2 \\ \uparrow \end{array}$$

$$\forall \alpha, \beta \quad rg = 1$$

$$\begin{cases} x = \gamma \\ y = \gamma^2 + 1 \\ z = -\gamma + 2 \end{cases}$$

$$\begin{cases} x = \alpha \cos\beta \\ y = \alpha \sin\beta \\ z = \alpha \end{cases}$$

$$rg \begin{pmatrix} \cos\beta & \sin\beta & 1 \\ -\alpha \sin\beta & \alpha\cos\beta & 0 \end{pmatrix} = \begin{cases} 2 & \forall \beta, \alpha \neq 0 \\ 1 & \forall \beta, \alpha = 0 \end{cases}$$

Trovare gli asintoti della curva

$$x^2y - 2xy^2 + 4xy - 2x + 1 = 0 \qquad \cdot X_0^3$$

$$F(X_0,X_1,X_2) = \left\{ \begin{array}{l} X_1^2X_2 - 2X_1X_2^2 + \cancel{4X_1X_2X_0} - \cancel{2X_1X_0^2} + \cancel{X_0^3} = 0 \\ X_0 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} X_1X_2(X_1 - 2X_2) = 0 \\ X_0 = 0 \end{array} \right.$$

$P_{1\infty} = (0,0,1)$ $\left\{ \begin{array}{l} X_1 = 0 \\ X_0 = 0 \end{array} \right.$

$P_{2\infty} = (0,1,0)$ $\left\{ \begin{array}{l} X_2 = 0 \\ X_0 = 0 \end{array} \right.$

$P_{3\infty} = (0,2,1)$ $\left\{ \begin{array}{l} X_1 = 2X_2 \\ X_0 = 0 \end{array} \right.$

$F_0 = 4X_1X_2 - 4X_1X_0 + 3X_0^2$

$F_1 = 2X_1X_2 - 2X_2^2 + 4X_2X_0 - 2X_0^2$

$F_2 = X_1^2 - 4X_1X_2 + 4X_1X_0$

	$P_{1\infty}$	$P_{2\infty}$	$P_{3\infty}$
F_0	0	0	8
F_1	-2	0	2
F_2	0	1	-4

$a_1: -2X_1 = 0$ $\qquad a_2: X_2 = 0$ $\qquad a_3: 8X_0 + 2X_1 - 4X_2 = 0$

$-2x = 0$ $\qquad y = 0$ $\qquad 8 + 2x - 4y = 0$

$$\begin{cases} x = u \\ y = v \\ z = F(u,v) \end{cases} \qquad \bar{P} \equiv (\bar{x}, \bar{y}, \bar{z}) = (\bar{u}, \bar{v}, F(\bar{u}, \bar{v}))$$

piano tang. in $\bar{P}$

$$\begin{vmatrix} (x-\bar{x}) & (y-\bar{y}) & (z-\bar{z}) \\ 1 & 0 & F'_u(\bar{u},\bar{v}) \\ 0 & 1 & F'_v(\bar{u},\bar{v}) \end{vmatrix}$$

$$(x-\bar{x}) \begin{vmatrix} 0 & F'_u \\ 1 & F'_v \end{vmatrix} - (y-\bar{y}) \begin{vmatrix} 1 & F'_u \\ 0 & F'_v \end{vmatrix} + (z-\bar{z}) \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$-F'_u(x-\bar{x}) - F'_v(y-\bar{y}) + (z-\bar{z}) = 0$$

$$(z-\bar{z}) = F'_u(\bar{u},\bar{v})(x-\bar{x}) + F'_v(\bar{u},\bar{v})(y-\bar{y}) = 0$$

$\Sigma : x^4 - y^2 - z^2 = 0$ Trovare il piano π tang. a Σ in

$F(x,y,z) =$

$P \equiv (3,9,0)$

	P
$F_x = 4x^3$	108
$F_y = -2y$	-18
$F_z = -2z$	0

$\pi : 108(x-3) - 18(y-9) + 0(z-0) = 0$

$$\boxed{108x - 18y - 162 = 0}$$

$$\mathcal{C} : \begin{cases} z = 0 \\ y = x^2 \end{cases}$$

Trovare il cono avente $\mathcal{C}$ come direttrice e vertice $V \equiv (0,0,1)$

$P_u \equiv (u, u^2, 0)$

$$r_u : \frac{x-0}{u-0} = \frac{y-0}{u^2-0} = \frac{z-1}{0-1}$$

$$\frac{x}{u} = \frac{y}{u^2} = \frac{z-1}{-1} = v$$

$$\begin{cases} x = uv \\ y = u^2 v \\ z = -v+1 \end{cases} \qquad \begin{cases} x = uv \\ y = u^2 v \\ v = 1-z \end{cases} \qquad \begin{cases} x = u(1-z) \\ y = u^2(1-z) \end{cases} \qquad \begin{cases} u = \frac{x}{1-z} \\ y = u^2(1-z) \end{cases}$$

$$y = \frac{x^2}{(1-z)^{\cancel{2}}} \cancel{(1-z)}$$

$$(1-z)y - x^2 = 0$$

$$\boxed{y - zy - x^2 = 0}$$

$\mathcal{C}: \begin{cases} z = 0 \\ y = x^2 \end{cases}$

Trovare il cilindro con direttrice $\mathcal{C}$ e generatrici $\parallel r: \begin{cases} x = 2y \\ z = y - 1 \end{cases}$

$\begin{cases} x = 2\alpha \\ y = \alpha \\ z = \alpha - 1 \end{cases}$ $(l, m, n) \sim (2, 1, 1)$

$P_u \equiv (u, u^2, 0)$

$$r_u : \frac{x-u}{2} = \frac{y-u^2}{1} = \frac{z-0}{1} = v$$

$\begin{cases} x = u + 2v \\ y = u^2 + v \\ z = v \end{cases}$ $\begin{cases} - \\ - \\ v = z \end{cases}$ $\begin{cases} x = u + 2z \\ y = u^2 + z \end{cases}$ $\begin{cases} u = x - 2z \\ y = u^2 + z \end{cases}$

$$\boxed{y = (x - 2z)^2 + z}$$