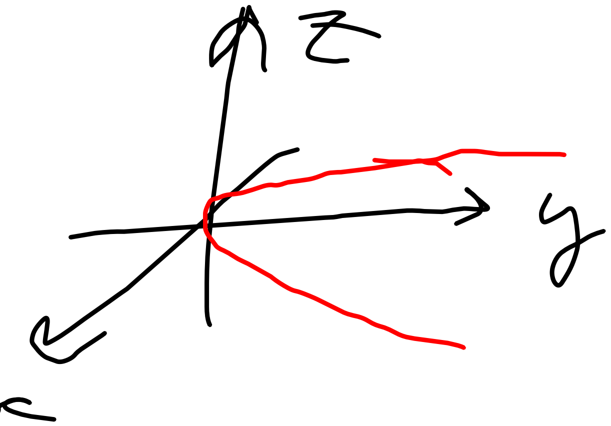
$\mathcal{C}$: $\begin{cases} z = 0 \\ y = x^2 \end{cases}$ Trovare la supf. Σ_1 di rotazione di $\mathcal{C}$ attorno all'asse x

Σ_1 : $\pm\sqrt{y^2+z^2} = x^2$ $\quad y^2+z^2 = x^4$



Σ_2 di rotaz. attorno all'asse y $\quad \Sigma_2$: $y = \left(\pm\sqrt{x^2+z^2}\right)^2$

$y = x^2+z^2$

$\mathcal{C}: \begin{cases} z=0 \\ y=x^2 \end{cases}$ Σ_3 di rotazione attorno a $r: \begin{cases} x=z \\ y=0 \end{cases}$ $(l,m,n) \sim (1,0,1)$

$\begin{cases} x=\beta \\ y=0 \\ z=\beta \end{cases}$

$\begin{cases} x=\alpha \\ y=\alpha^2 \\ z=0 \end{cases}$ $P_\alpha: (\alpha, \alpha^2, 0)$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Π_α per P_α, $\perp r$ $\quad 1(x-\alpha)+0(y-\alpha^2)+1(z-0)=0$

$\Pi_\alpha: x+z-\alpha=0$

sfera $\mathcal{S}_\alpha$ con centro $O\equiv(0,0,0)$ e passante per P_α

$(x-0)^2+(y-0)^2+(z-0)^2=(\alpha-0)^2+(\alpha^2-0)^2+(0-0)^2$

$\Sigma_3: \begin{cases} \Pi_\alpha: x+z-\alpha=0 \\ \mathcal{S}_\alpha: x^2+y^2+z^2=\alpha^2+\alpha^4 \end{cases}$

$\begin{cases} \alpha=x+z \\ \text{———} \end{cases}$ $\quad x^2+y^2+z^2=(x+z)^2+(x+z)^4$

$\cancel{x^2}+y^2+\cancel{z^2}=\cancel{x^2}+2xz+\cancel{z^2}+x^4+4x^3z+6x^2z^2+4xz^3+z^4$

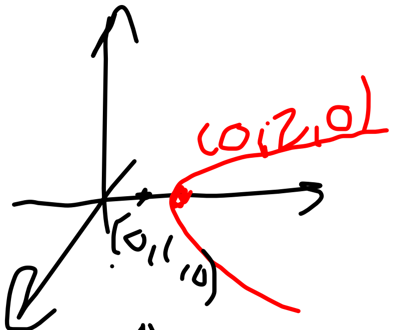
$\mathcal{C} : \begin{cases} z = 0 \\ y = x^2 \end{cases}$ $\quad \Sigma_4$ di rotazione attorno all'asse z $\begin{cases} x = 0 \\ y = 0 \end{cases}$

$P_\alpha \equiv (\alpha, \alpha^2, 0)$

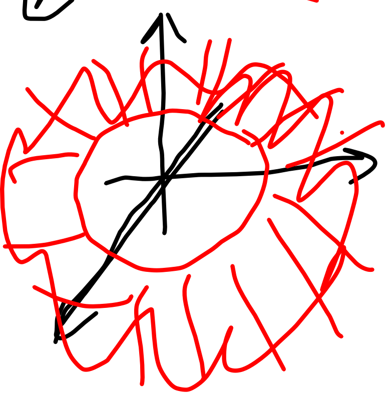
Π_α per P_α $\perp$ asse z $\quad \Pi_\alpha : 0(x-\alpha) + 0(y-\alpha^2) + 1(z-0) = 0$ $\quad (l, m, n) \sim (0, 0, 1)$

$\Pi_\alpha : \begin{cases} z = 0 \\ x^2 + y^2 + z^2 = \alpha^2 + \alpha^4 \end{cases} \longrightarrow z = 0$

$\mathcal{S}_\alpha$ sfera per P_α e centro O $\quad \Sigma_4 \quad \mathcal{S}_\alpha :$



$$C : \begin{cases} z = 0 \\ y = x^2 + 2 \end{cases} \qquad P_\alpha = (\alpha, \alpha^2+2, 0)$$



$$\Pi_\alpha : z = 0$$

$$S_\alpha : (x-0)^2 + (y-0)^2 + (z-0)^2 = (\alpha - 0)^2 + (\alpha^2+2-0)^2 + (0-0)^2$$

$$x^2 + y^2 + z^2 = \alpha^2 + \alpha^4 + 4\alpha^2 + 4$$

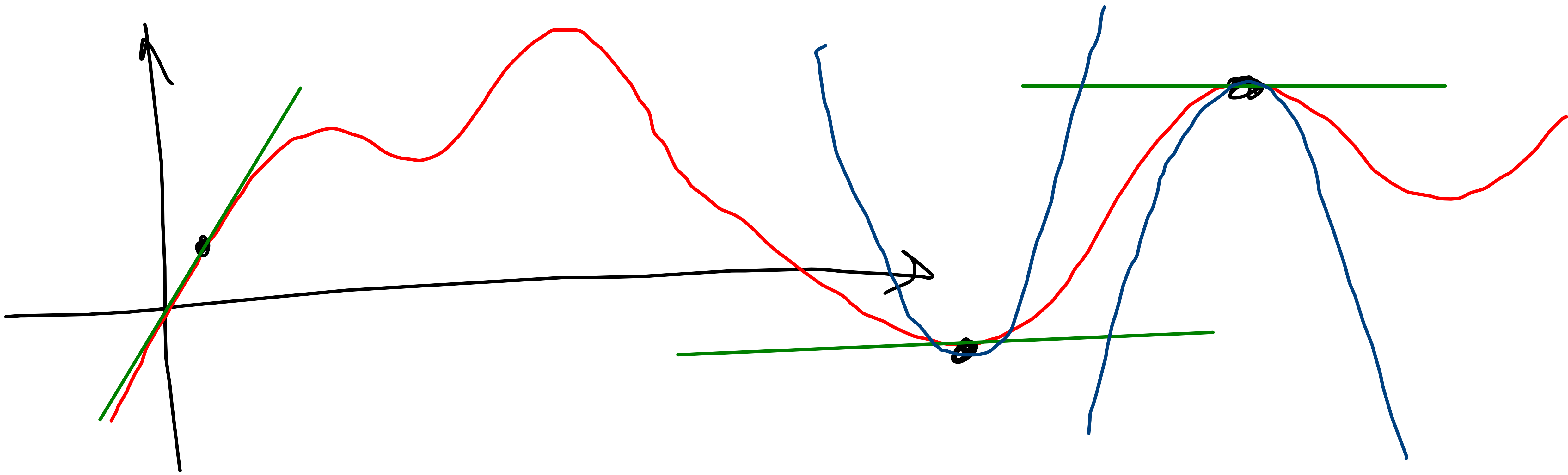
$$\Sigma_\alpha \begin{cases} x^2 + y^2 + z^2 = \alpha^4 + 5\alpha^2 + 4 \\ z = 0 \end{cases} \qquad z = 0 \; ?$$

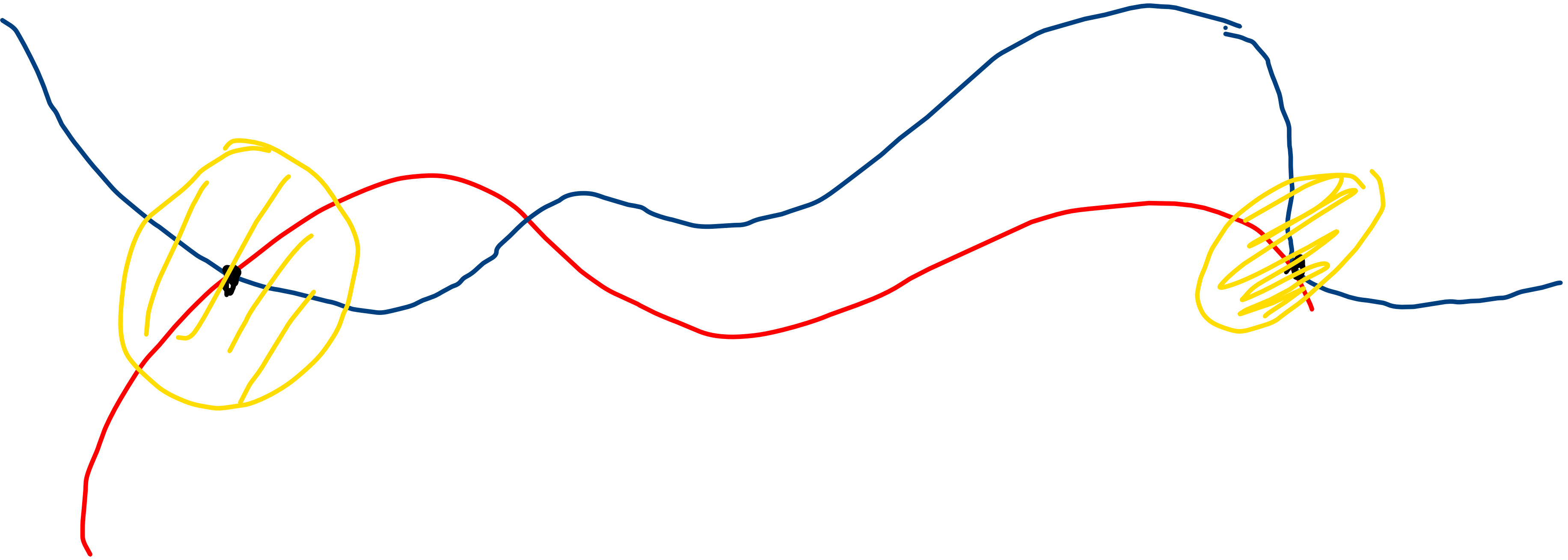
$$(0,1,0) \qquad \begin{cases} 0 + 1 + 0 = \alpha^4 + 5\alpha^2 + 4 \\ 0 = 0 \end{cases}$$

$$\alpha^4 + 5\alpha^2 + 3 = 0$$

$$\alpha^2 = \frac{-5 \pm \sqrt{25 - 12}}{2} = \frac{-5 \pm \sqrt{13}}{2} = \frac{-5 - \sqrt{13}}{2} < 0 \qquad \frac{-5 + \sqrt{13}}{2} < 0$$

$$\alpha = \pm \sqrt{\frac{-5-\sqrt{13}}{2}} \qquad \pm \sqrt{\frac{-5 \pm \sqrt{13}}{2}}$$





e_1 e e_2 si attraversano nel punto comune P

significa:

$\exists$ disco del piano centrato in P tale che

1) e_1 separa il disco in esattamente due parti

e 2) e_1 ha punti sia in una sia nell'altra parte

---

non si attraversano:

$\exists$ disco - - - - - tale che

1) ———

e 2) i punti di e_1 contenuti in tale disco sono tutti nella stessa parte delle due determinate da e_1

$C_1 : y = f(x) \quad f(0) = 0$

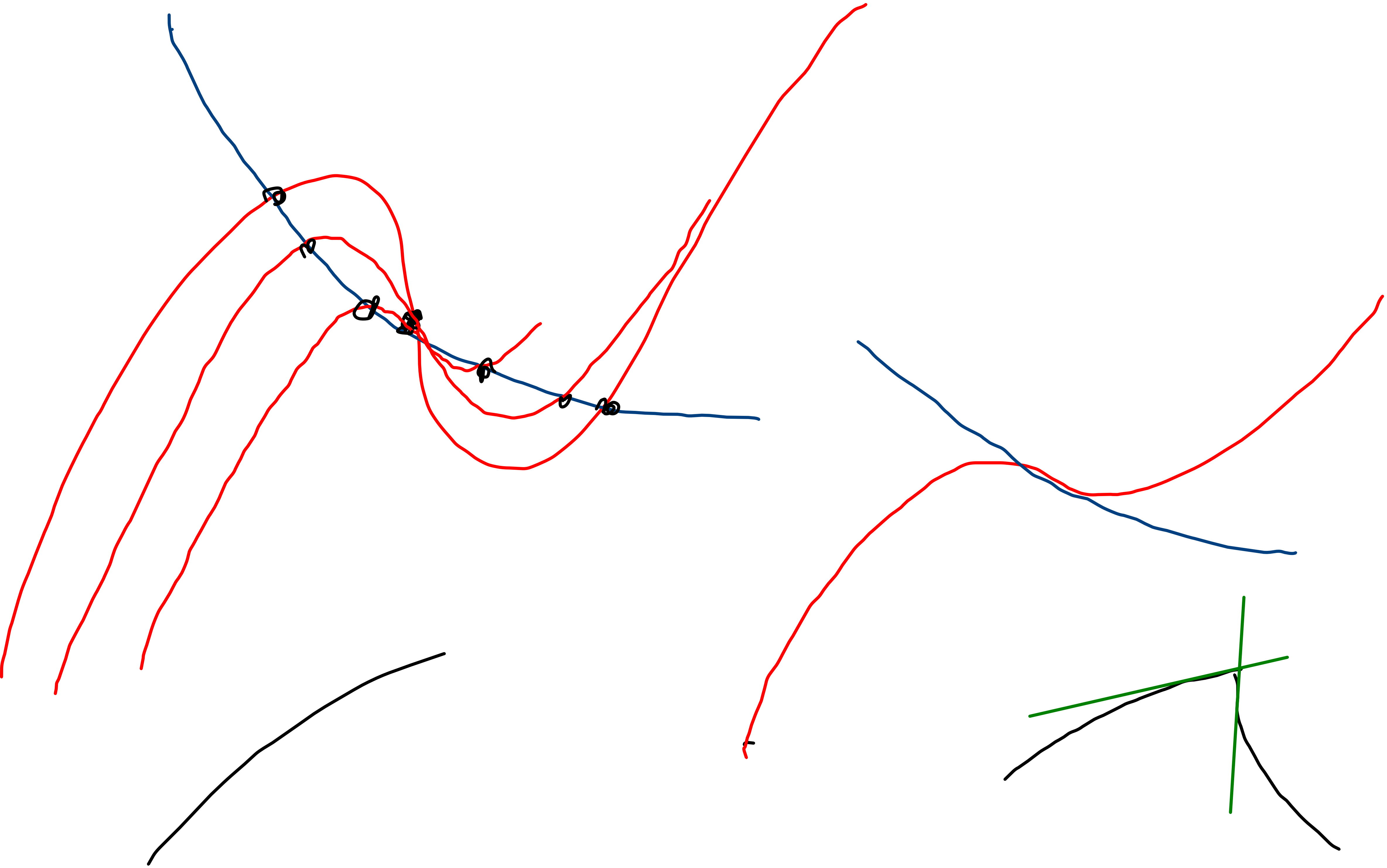
$C_2 : y = \varphi(x) \quad \varphi(0) = 0$

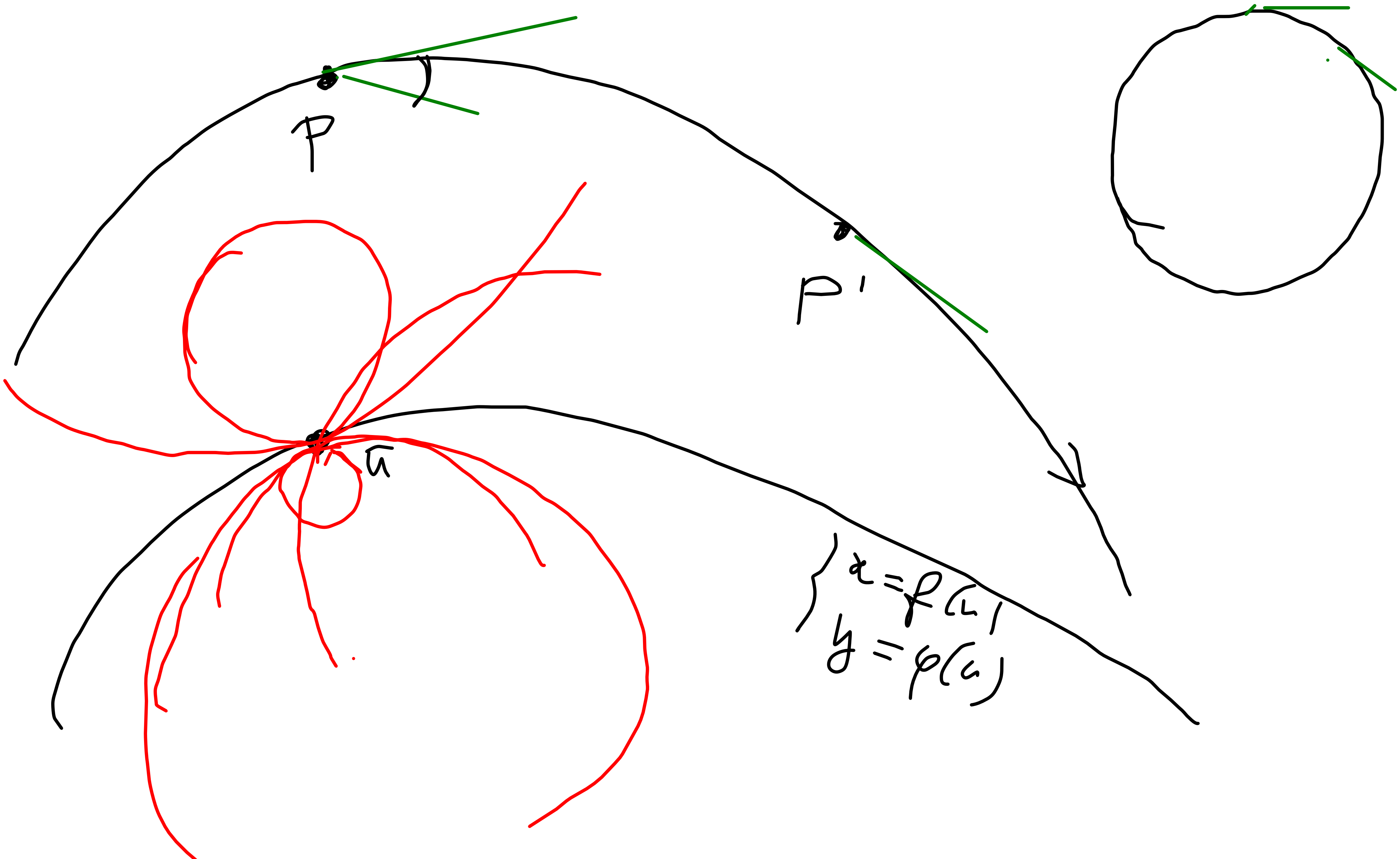
$C_1 : y_1 = a_1 x + a_2 x^2 + \cdots + a_n x^n + a_{n+1} x^{n+1} + \text{———}$

$C_2 : y_2 = b_1 x + b_2 x^2 + \cdots + b_n x^n + b_{n+1} x^{n+1} + \text{———}$

$$y_1 - y_2 = \underbrace{(a_1 - b_1)}_{=0} x + \underbrace{(a_2 - b_2)}_{=0} x^2 + \cdots + \underbrace{(a_n - b_n)}_{=0} x^n + \underbrace{(a_{n+1} - b_{n+1})}_{\neq 0} x^{n+1} + \text{———}$$

$$y_1 - y_2 = \underset{\neq 0}{k}\, x^{n+1}$$




P
P'
$\bar{u}$
$x = f(u)$
$y = \varphi(u)$

$C: y = 4x^2 \quad P \equiv (1,4)$

$\Gamma: x^2+y^2+ax+by+c=0$

$$\begin{cases} x = u \\ y = 4u^2 \end{cases} \qquad P \Leftrightarrow u = 1$$

$\Phi(u) = u^2 + 16u^4 + au + 4bu^2 + c$

$\Phi'(u) = 2u + 64u^3 + a + 8bu$

$\Phi''(u) = 2 + 192u^2 + 8b$

$\Phi'''(u) = 384u$

Trovare la circ. osculatrice in P e calcolarne l'ordine di contatto

Impongo

$$\begin{cases} \Phi(1) = 0 \\ \Phi'(1) = 0 \\ \Phi''(1) = 0 \end{cases} \qquad \begin{cases} 17 + a + 4b + c = 0 \\ 66 + a + 8b = 0 \\ 194 + 8b = 0 \end{cases} \qquad \begin{cases} c = -128 + 97 - 17 = -48 \\ a = 194 - 66 = 128 \\ b = -\frac{97}{4} \end{cases}$$

$x^2 + y^2 + 128x - \frac{97}{4}y - 48 = 0$

$\Phi'''(1) = 384 \neq 0$

ordine $= 2$

$C: \; y = 4x^2 \qquad P \equiv (0,0) \qquad \begin{cases} x = u \\ y = 4u^2 \end{cases} \qquad u = 0$

$$\Phi(u) = u^2 + 16u^4 + au + 4bu^2 + c$$

$$\Phi'(u) = 2u + 64u^3 + a + 8bu$$

$$\Phi''(u) = 2 + 192u^2 + 8b$$

$$\Phi'''(u) = 384u$$

$$\Phi^{IV}(u) = 384$$

$$\begin{cases} \Phi(0) = 0 \\ \Phi'(0) = 0 \\ \Phi''(0) = 0 \end{cases} \qquad \begin{cases} c = 0 \\ a = 0 \\ 8b + 2 = 0 \end{cases} \qquad \begin{cases} a = 0 \\ b = -\frac{1}{4} \\ c = 0 \end{cases}$$

$$x^2 + y^2 - \frac{1}{4}y = 0$$

$$\Phi'''(0) = 0 \qquad \Phi^{IV}(0) = 384 \neq 0$$

ordine: 3

Centro $(\bar{x}, \bar{y})$ raggio $= R$

$$(x-\bar{x})^2 + (y-\bar{y})^2 - R^2 = 0$$

$$x^2 - 2\bar{x}x + \bar{x}^2 + y^2 - 2\bar{y}y + \bar{y}^2 - R^2 = 0$$

$$x^2 + y^2 \underbrace{-2\bar{x}}_{a}\,x \underbrace{-2\bar{y}}_{b}\,y + \underbrace{(\bar{x}^2+\bar{y}^2-R^2)}_{c} = 0$$

$$\begin{cases} -2\bar{x} = 0 \\ -2\bar{y} = -\dfrac{1}{4} \\ \bar{x}^2+\bar{y}^2-R^2 = 0 \end{cases}$$

$\bar{x} = 0$

$\bar{y} = \dfrac{1}{8}$

$R^2 = \bar{x}^2 + \bar{y}^2 = \dfrac{1}{64}$ $\qquad R = \dfrac{1}{8}$