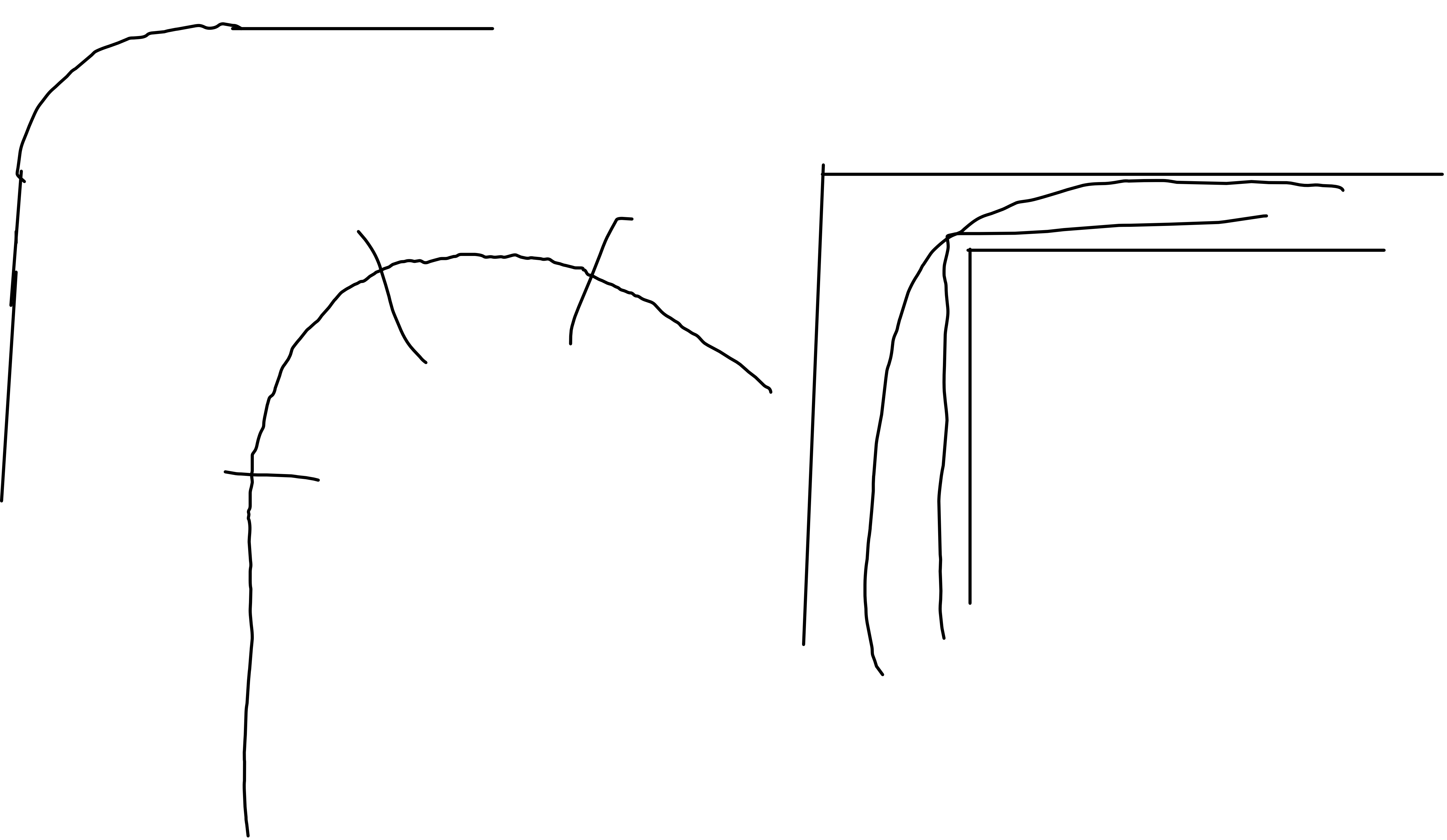
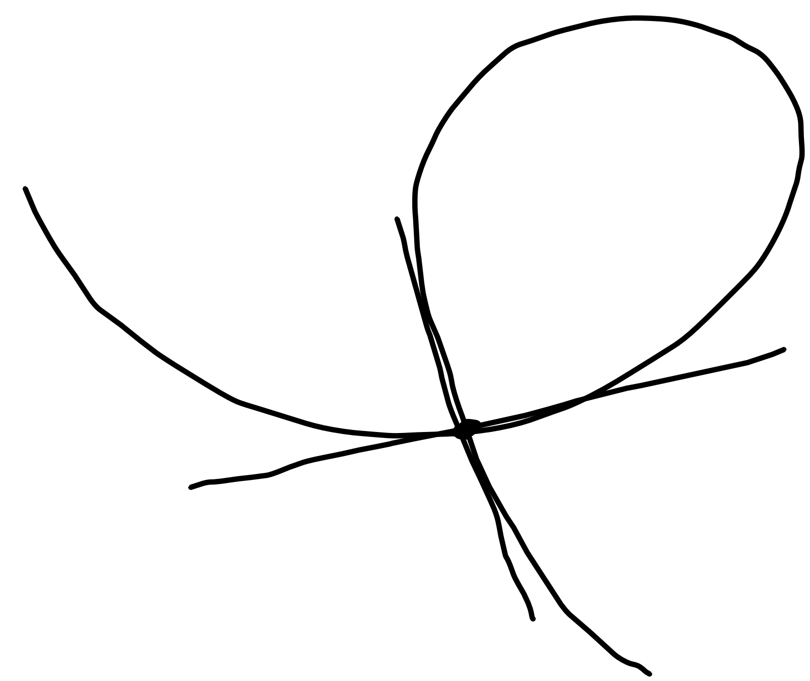


$$\begin{cases} x = f(u) \\ y = \varphi(u) \end{cases} \qquad (\bar{x}, \bar{y}) \longleftrightarrow \bar{u}$$

tang.: $$\frac{x - f(\bar{u})}{f'(\bar{u})} = \frac{y - \varphi(\bar{u})}{\varphi'(\bar{u})}$$



$$\begin{cases} x = u^4 - 5u^2 + 4 \\ y = u^3 - u \end{cases}$$

$$\begin{cases} u^4 - 5u^2 + 4 = 0 \\ u^3 - u = 0 \end{cases}$$

$u^2 = \dfrac{5 \pm \sqrt{25-16}}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$\cancel{u = \pm 2}$

$u = \pm 1$

	$u=-1$	$u=1$
$x' = 4u^3 - 10u$	6	-6
$y' = 3u^2 - 1$	2	2

$u = 2 \quad 8 - 2 \neq 0$

$u = -2 \quad -8 + 2 \neq 0$

$u = -1 \quad \dfrac{x}{6} = \dfrac{y}{2}$

$u = +1 \quad \dfrac{x}{-6} = \dfrac{y}{2}$

$\mathcal{C}: f(x,y)=0 \quad P_0 \equiv (x_0, y_0) \in \mathcal{C} \quad f(x_0,y_0)=0$

Indicherò con

$$\left((x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y} \right) f(x_0,y_0)$$

l'espressione

$$\left((x-x_0)\frac{\partial f}{\partial x}(x_0,y_0) + (y-y_0)\frac{\partial f}{\partial x}(x_0,y_0) \right)$$

e, analogamente,

$$\left((x-x_0)\frac{\partial}{\partial x}+(y-y_0)\frac{\partial}{\partial y}\right)^2 f(x_0,y_0)$$

sta per

$$\left((x-x_0)^2\frac{\partial^2}{\partial x^2}f(x_0,y_0)+2(x-x_0)(y-y_0)\frac{\partial^2 f}{\partial x\partial y}(x_0,y_0)+(y-y_0)^2\frac{\partial^2 f}{\partial y^2}(x_0,y_0)\right)$$

e così via

$$
\begin{cases}
0 = f(x,y) = \left((x-x_0)\dfrac{\partial}{\partial x} + \overset{k(x-x_0)}{(y-y_0)}\dfrac{\partial}{\partial y} \right) f(x_0,y_0) + \\[2ex]
\qquad + \dfrac{1}{2}\left((x-x_0)\dfrac{\partial}{\partial x} + \overset{k(x-x_0)}{(y-y_0)}\dfrac{\partial}{\partial y} \right)^2 f(x_0,y_0) + \\[1ex]
\qquad \vdots \\[1ex]
\qquad + \dfrac{1}{(s-1)!}\left((x-x_0)\dfrac{\partial}{\partial x} + \overset{k(x-x_0)}{(y-y_0)}\dfrac{\partial}{\partial y} \right)^{s-1} f(x_0,y_0) + \\[2ex]
\qquad + \dfrac{1}{s!}\left((x-x_0)\dfrac{\partial}{\partial x} + \overset{k(x-x_0)}{(y-y_0)}\dfrac{\partial}{\partial y} \right)^{s} f(x_0,y_0) + \\[2ex]
\qquad + \dfrac{1}{(s+1)!}\left((x-x_0)\dfrac{\partial}{\partial x} + \overset{k(x-x_0)}{(y-y_0)}\dfrac{\partial}{\partial y} \right)^{s+1} f(x_0,y_0) + \\[1ex]
\qquad + \cdots \\[2ex]
y - y_0 = k(x-x_0)
\end{cases}
$$

$$\begin{cases} 0 = f(x,y) \\ y - y_0 = k(x - x_0) \end{cases}$$

$$(x-x_0)\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right) f(x_0,y_0) +$$

$$+ \frac{(x-x_0)^2}{2}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(x_0,y_0) +$$

$$\vdots$$

$$+ \frac{(x-x_0)^{s-1}}{(s-1)!}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{s-1} f(x_0,y_0) +$$

la generica retta per P_0 ha in P_0 s intersezioni $= 0 \ \forall k$

$$+ \frac{(x-x_0)^{s}}{s!}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{s} f(x_0,y_0) +$$

le particolari rette che hanno in P_0 almeno $s+1$ intersezioni sono quelle il cui le annulla

$$+ \frac{(x-x_0)^{s+1}}{(s+1)!}\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^{s+1} f(x_0,y_0) +$$

$$+ \ldots$$

Esempio 6 n. 4

$\mathcal{C}: F(x,y) = x^2y - xy^2 - x + y = 0$

trovarne i punti multipli reali e le tangenti in essi

$F_x = 2xy - y^2 - 1$

$F_y = x^2 - 2xy + 1$

$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \end{cases}$ $\begin{cases} F = 0 \\ 2xy - 1 = y^2 \\ 2xy - 1 = +x^2 \end{cases}$ $\begin{cases} F = 0 \\ 2xy - 1 = y^2 \\ x^2 = y^2 \end{cases}$ $\begin{cases} F = 0 \\ 2xy - 1 = y^2 \\ x = \pm y \end{cases}$ 1 $\cancel{2}$

1 $\begin{cases} F = 0 \\ 2y^2 - 1 = y^2 \\ x = y \end{cases}$ $\begin{cases} F = 0 \\ y^2 = 1 \\ x = y \end{cases}$ $\begin{cases} y = 1 & 3 \\ y = -1 & 4 \end{cases}$

3 $\begin{cases} 1 - 1 - 1 + 1 = 0 \\ y = 1 \\ x = y = 1 \end{cases}$ $(1,1)$

4 $\begin{cases} -1 + 1 + 1 - 1 = 0 \\ y = -1 \\ x = y = -1 \end{cases}$ $(-1,-1)$

2 $\begin{cases} F = 0 \\ -2y^2 - 1 = y^2 \\ x = -y \end{cases}$ $\cancel{\begin{cases} F = 0 \\ 3y^2 = -1 \\ x = -y \end{cases}}$

$F_x = 2xy - y^2 - 1$

$F_y = x^2 - 2xy + 1$

		$(1,1)$	$(-1,-1)$
$F_{xx} = 2y$		2	-2
$F_{xy} = 2x - 2y$		0	0
$F_{yy} = -2x$		-2	2

$\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(1,1) = 0 \qquad 2 + 2\cdot 0\cdot k - 2k^2 = 0 \qquad k^2 = 1 \quad k = \pm 1$

$y - 1 = 1(x-1)$

$y - 1 = -1(x-1)$

$\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 f(-1,-1) = 0 \qquad -2 + 2\cdot 0\cdot k + 2k^2 = 0$

$y + 1 = 1(x+1)$

$y + 1 = -1(x+1)$

Se voglio il complesso delle tangenti in $(1,1)$

$$\left((x-1)\frac{\partial}{\partial x} + (y-1)\frac{\partial}{\partial y}\right)^2 f(1,1) = 0$$

$$2 \cdot (x-1)^2 + 2 \cdot 0 \cdot (x-1)(y-1) - 2(y-1)^2 = 0$$

$$(x-1)^2 = (y-1)^2$$

$7xy^2$

$C: x^5 - y^4 + 7x^2y = 0$

$F(x,y) =$

$O \equiv (0,0)$ è multiplo di molt. 3

	$(0,0)$
$F_x = 5x^4 + 14xy$	0
$F_y = -4y^3 + 7x^2$	0
$F_{xx} = 20x^3 + 14y$	0
$F_{xy} = 14x$	0
$F_{yy} = -12y^2$	0

	$(0,0)$
$F_{xxx} = 60x^2$	0
$F_{xxy} = 14 + 0$	14
$F_{xyy} = 0$	0
$F_{yyy} = -24y$	0

$$\left(\frac{\partial}{\partial x}+k\frac{\partial}{\partial y}\right)^3 f(0,0)=0$$

$$0+3\cdot 14k+3\cdot 0\cdot k^2+0k^3=0$$

$42k=0$

$k=0$

$k=\infty$ 2 volte

$$\left(a\frac{\partial}{\partial x}+b\frac{\partial}{\partial y}\right)^3 f(0,0)=0$$

$y=kx$

$by=ax$

$$0a^3+3\cdot 14a^2b+3\cdot 0\cdot ab^2+0\cdot b^3=0$$

$3\cdot 14a^2b=0$

2 volte $a=0$

$b=0$