Trovare, mediante la nozione di contatto, il piano tangente a $\Sigma: x^3 - 2y^2 - 2z - 4x + 4 = 0$ in $P = (2,0,2)$

Generico piano per P:

$$z - 2 = a(x-2) + b(y-0)$$

$$\begin{cases} x = u+2 \\ y = v \\ z = 2 + au + bv \end{cases}$$

$P \leftrightarrow \begin{matrix} u = 0 \\ v \end{matrix}$

in $(u,v) = (0,0)$

$$\Phi(u,v) = -2v^2 - 2bv + u^3 + 6u^2 - 2au + 8u$$

$$\Phi_u = 3u^2 + 12u - 2a + 8$$

$$\Phi_v = -4v - 2b$$

$$\begin{cases} \Phi_u(0,0) = 0 \\ \Phi_v(0,0) = 0 \end{cases} \quad \begin{cases} -2a + 8 = 0 \\ -2b = 0 \end{cases} \quad \begin{cases} a = 4 \\ b = 0 \end{cases}$$

$$z - 2 = 4(x-2) + 0(y-0)$$

$$4x - z - 6 = 0$$

Trovare gli eventuali punti multipli e le tangenti in essi alla curva $C: \begin{cases} x = u^4 - 5u^2 + 4 \\ y = u^3 - u \end{cases}$

$u^4 - 5u^2 + 4 - x = 0$

$u^3 - u - y = 0$

Divisioni successive, ultimo resta

$16y^4 - 176xy^2 - 576y^2 - 16x^3 + 64x^2 = 0$

$F(x,y)$

$$\begin{cases} F = 0 \\ F_x = 0 \\ F_y = 0 \end{cases}$$

$y^4 - 8xy^2 + 16x^2 = (y^2 - 4x)^2$

$F_x = -176y^2 - 48x^2 + 128x$

$F_y = 64y^3 - 352xy - 1152y = 32y(2y^2 - 11x - 36)$

$y = 0$

$2y^2 - 11x - 36 = 0$

$$\left(16y^4 - 176xy^2 - 576y^2 - 16x^3 + 64x^2\right) = 0 \qquad F_y = 0$$

$$F(x,y) \qquad y^4 - 8xy^2 + 16x^2 = (y^2-4x)^2$$

$$F_x = -176y^2 - 48x^2 + 128x$$

$$F_y = 64y^3 - 352xy - 1152y = 32y(2y^2 - 11x - 36)$$

$y = 0$ (1)

$2y^2 - 11x - 36 = 0$ (2)

(1) $\begin{cases} F = 0 \\ -48x^2 + 128x = 0 \\ y = 0 \end{cases}$ $\quad -16x(3x-8) = 0$

$x = 0$ (3)

$x = \frac{8}{3}$ (4)

(3) $\begin{cases} 0 = 0 \\ x = 0 \\ y = 0 \end{cases}$ $\quad (0,0)$

(4) $\begin{cases} F \neq 0 \\ x = \frac{8}{3} \\ y = 0 \end{cases}$

(2) $\begin{cases} F = 0 \\ F_x = 0 \\ y^2 = \frac{11x+36}{2} \end{cases}$ $\quad \begin{cases} F = 0 \\ -48x^2 - 840x - 3168 = 0 \\ y^2 = \_\_\_ \end{cases}$ $\quad x = \cancel{-12}, \ \cancel{-\frac{11}{2}}$

$x = -12$

$y^2 = \frac{-12 \cdot 11 + 36}{2} < 0$

$x = -\frac{11}{2}$

$F \neq 0$

$$-576y^2 + 64x^2 = 0$$

$$-64(3y-x)(3y+x) = 0 \qquad \boxed{\begin{matrix} 3y - x = 0 \\ 3y + x = 0 \end{matrix}}$$

$$\left(16y^4 - 176xy^2 - 576y^2 - 16x^3 + 64x^2\right) = 0 \qquad F_y = 0 \qquad (0,0)$$

$$F(x,y) \parallel \quad y^4 - 8xy^2 + 16x^2 = (y^2-4x)^2$$

$$F_x = -176y^2 - 48x^2 + 128x \qquad 0$$

$$F_y = 64y^3 - 352xy - 1152y = 32y(2y^2 - 11x - 36) \quad y = 0, \quad 2y^2 - 11x - 36 = 0 \qquad 0$$

$$F_{xx} = 128 - 96x \qquad 128$$

$$F_{xy} = -352y \qquad 0$$

$$F_{yy} = 192y^2 - 352x - 1152 \qquad -1152$$

$$\left(\frac{\partial}{\partial x} + k\frac{\partial}{\partial y}\right)^2 F(0,0) = 0 \qquad 128 + 0k - 1152k^2 = 0 \quad k = \pm\frac{1}{3}$$

$$y = \frac{1}{3}x$$

$$y = -\frac{1}{3}x$$

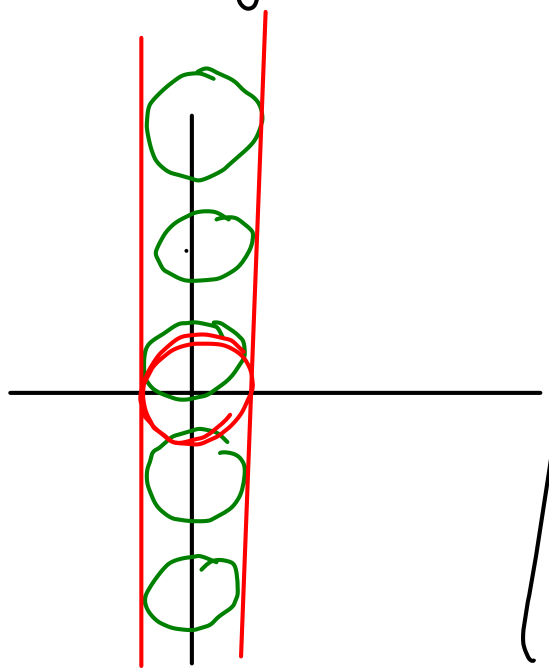
$x^2+(y-\alpha)^2-1=0$

$F(x,y,\alpha)= x^2+y^2-2\alpha y+\alpha^2-1=0$

$\begin{cases} F=0 \\ F_\alpha=0 \end{cases}$ $\begin{cases} F=0 \\ \alpha=y \end{cases}$

$F_\alpha=-2y+2\alpha$

$x^2+\cancel{y^2}-2\cancel{y^2}+\cancel{y^2}-1=0$ $x^2-1=0$



$x^2+(y-\alpha^2)^2-1=0$

$F(x,y,\alpha)= x^2+y^2-2\alpha^2 y+\alpha^4-1=0$ $F_\alpha=-4\alpha y+4\alpha^3$

$\begin{cases} F=0 \\ 4\alpha(\alpha^2-y)=0 \end{cases}$ ① $\alpha=0$ ② $\alpha^2=y$

① $x^2+y^2-1=0$

$\cup$

② $x^2+\cancel{y^2}-2\cancel{y^2}+\cancel{y^2}-1=0$

$C_\alpha : y = (x-\alpha)^2 \qquad y = x^2 - 2\alpha x + \alpha^2$

$F(x,y;\alpha) = \alpha^2 - 2x\alpha + x^2 - y \qquad \begin{cases} F = 0 \\ F_\alpha = 0 \end{cases} \qquad \begin{cases} F = 0 \\ \alpha = x \end{cases}$

$F_\alpha = 2\alpha - 2x$

$\cancel{x^2} - 2\cancel{x^2} + \cancel{x^2} - y = 0 \qquad y = 0$

$$(y-(x-a)^2)(x-a-1)=0$$

$$F = xy - ay - y - x^3 + 3ax^2 + x^2 - 3a^2x - 2ax + a^3 + a^2$$

$$\bar{F} = 3F - a \cdot F_a$$

$$F_a = -y + 3x^2 - 6ax - 2x + 3a^2 + 2a$$

$$\bar{F} = (1-3x)a^2 + (6x^2 - 4x - 2y)a + 3xy - 3y - 3x^3 + 3x^2$$

Divisioni successive fra $\bar{F}$ ed F_a : ultimo resto

$$\frac{-9(y-1)^2 y}{(3y+1)^2}$$

unione di $y=1$ e $y=0$

Eulero : $-12y(y^2-2y+1)$

Trovare l'evoluta di $\mathcal{C}$, $y=x^2$ $\qquad \begin{cases} x=u \\ y=u^2 \end{cases}$ $\quad P_u \equiv (u,u^2)$

$t_u: \; y-u^2=2u(x-u) \qquad 2ux-y-u^2=0$

$n_u: \; \dfrac{x-u}{2u}=\dfrac{y-u^2}{-1} \qquad -(x-u)=2u(y-u^2)$

$-x+u=2uy-2u^3$

$2u^3+(1-2y)u-x=0$

$F(x,y,u)=$

$F_u=6u^2+1-2y$

$\tilde{F}=3F-uF_u=$

$=6u^3+3(1-2y)u-3x$

$-6u^3-(1-2y)u$

$= \; // \; 2(1-2y)u-3x$

Risultante di $\begin{cases} F_u=0 \\ \tilde{F}=0 \end{cases}$

$$\begin{vmatrix} 6 & 0 & (1-2y) \\ 2(1-2y) & -3x & 0 \\ 0 & 2(1-2y) & -3x \end{vmatrix} = \cancel{-32xy^2+16y^2+32xy-16y+54x^2-8x+4} = 0$$

$=(2-4y)^2(1-2y)+54x^2=$

$=-32y^3+48y^2-24y+54x^2+4$ (OK)

ERRORE? SI

$$\begin{cases} x = u \\ y = u^2 \end{cases} \qquad F(x,y) = x^2 + y^2 + ax + by + c = 0$$

$$\Phi(u) = u^2 + u^4 + au + bu^2 + c$$

$$\Phi'(u) = 2u + 4u^3 + a + 2bu = 0$$

$$\Phi''(u) = 2 + 12u^2 + 2b = 0$$

$$\begin{cases} 2u + 4u^3 + a - 2u - 12u^3 = 0 \\ b = -1 - 6u^2 \end{cases}$$

$$\begin{cases} a = 8u^3 \\ b = -1 - 6u^2 \end{cases} \qquad C_u = \left(-4u^3, \frac{1+6u^2}{2}\right)$$

$$\mathscr{E} : \begin{cases} x = -4u^3 \\ y = \frac{1}{2} + 3u^2 \end{cases}$$

$$4u^3 + x = 0$$

$$6u^2 - 2y + 1 = 0$$

$$\begin{vmatrix} 4 & 0 & 0 & x & 0 \\ 0 & 4 & 0 & 0 & x \\ 6 & 0 & 1-2y & 0 & 0 \\ 0 & 6 & 0 & 1-2y & 0 \\ 0 & 0 & 6 & 0 & 1-2y \end{vmatrix} = 16(1-2y)^3 + 216x^2 =$$

$$= -128y^3 + 192y^2 - 96y + 216x^2 + 16$$