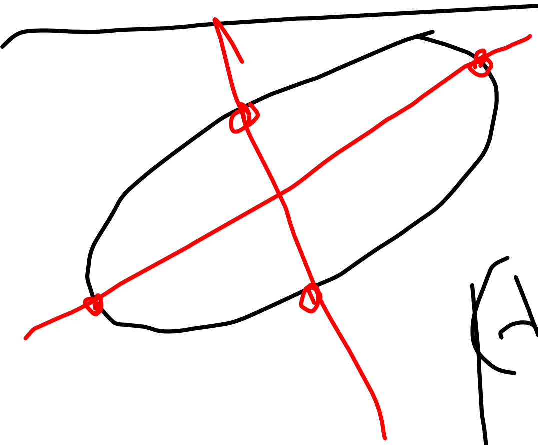
Data l'ellisse $\mathcal{E}: 5x^2+5y^2+6xy+10x+6y-11=0$
scrivere l'eq. del fascio di coniche passanti per i vertici di $\mathcal{E}$.



$$A=\begin{pmatrix}-11 & 5 & 3\\ 5 & 5 & 3\\ 3 & 3 & 5\end{pmatrix}\qquad M_0^0=\begin{pmatrix}5 & 3\\ 3 & 5\end{pmatrix}$$

$$\begin{vmatrix}(\lambda-5) & -3\\ -3 & (\lambda-5)\end{vmatrix}=(\lambda-5)^2-(-3)^2=(\lambda-\cancel{5}-3)(\lambda-5+3)=$$
$$=(\lambda-8)(\lambda-2)$$

$$U_8:\begin{pmatrix}3 & -3\\ \cancel{-3} & \cancel{3}\end{pmatrix}\begin{pmatrix}l\\ m\end{pmatrix}=\begin{pmatrix}0\\ \cancel{0}\end{pmatrix}\qquad \begin{matrix}3l-3m=0\\ l=m\end{matrix}\qquad \text{scelgo } m=1\qquad P_{1\infty}=(0,1,1)$$

$$U_2:\begin{pmatrix}-3 & -3\\ \cancel{-3} & \cancel{-3}\end{pmatrix}\begin{pmatrix}l\\ m\end{pmatrix}=\begin{pmatrix}0\\ \cancel{0}\end{pmatrix}\qquad \begin{matrix}-3l-3m=0\\ l=-m\end{matrix}\qquad \text{scelgo } m=1\qquad P_{2\infty}=(0,-1,1)$$

$$\begin{matrix}(0\ 1\ 1)\\ (0\ -1\ 1)\end{matrix}\cdot A\cdot\begin{pmatrix}1\\ x\\ y\end{pmatrix}=0\qquad \begin{matrix}8+8x+8y=0 & x+y+1=0\\ -2-2x+2y=0 & x-y+1=0\end{matrix}$$

$$\mathcal{F}: \alpha(5x^2+5y^2+6xy+10x+6y-11)+\beta(x+y+1)(x-y+1)=0$$

$$A = \begin{pmatrix} -11 & 5 & 3 \\ 5 & 5 & 3 \\ 3 & 3 & 5 \end{pmatrix}$$

Metodo alternativo per trovare gli assi

Generico punto improprio $P_\infty \equiv (0, l, m)$

Sua polare: $(0 \; l \; m) \cdot A \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$ $\quad \underbrace{(5l+3m)}_{c} + \underbrace{(5l+3m)}_{a}x + \underbrace{(3l+5m)}_{b}y = 0$

Condizione di $\perp$: $(a, b) \sim (l, m)$ cioè

$$(5l+3m, 3l+5m) \sim (l, m)$$

$$\cancel{5l}m + 3m^2 = 3l^2 + \cancel{5lm}$$

$$m^2 = l^2$$

$$m = \pm l \qquad \text{scelgo } l = 1$$

$$P_{1\infty} \equiv (0, 1, 1) \quad - - - -$$

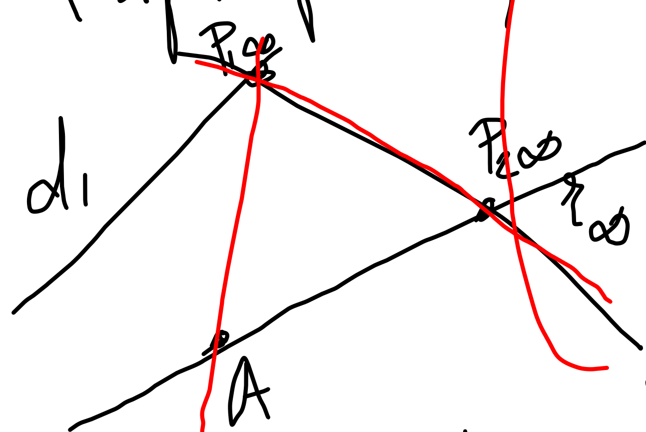
$$P_{2\infty} \equiv (0, 1, -1)$$

Dati $d_1: x+2y-6=0$ $\quad d_2: 2x-y=0$ $\quad A\equiv(1,1)$ trovare il fascio $\mathcal{F}$ di iperboli per A, aventi d_1 come asintoto e seconda asintoto $\parallel d_2$

d_1 è tangente a tutte le coniche di $\mathcal{F}$ in $P_{1\infty}\equiv(0,2,-1)$ punto improprio di d_1; tutte passano per A, il secondo asintoto di ogni iperbole di $\mathcal{F}$ ha come punto improprio quello di d_2: $P_{2\infty}=(0,1,2)$

$$\boxed{\alpha(x+2y-6)(2x-y-1)+\beta(x+2y-3)=0}$$

$$\begin{cases} X_1+2X_2-6X_0=0 \\ X_0=0 \end{cases} \qquad \begin{cases} X_1+2X_2=0 \\ X_0=0 \end{cases} \qquad \begin{cases} X_1=-2X_2 \\ X_0=0 \end{cases}$$



$\Gamma_1 = d_1 \cup AP_{2\infty}$

$\Gamma_2 = AP_{1\infty} \cup \underbrace{P_{1\infty}P_{2\infty}}_{r_\infty}$

$\mathcal{F}: \alpha(X_1+2X_2-6X_0)(2X_1-X_2-X_0)+\beta(X_1+2X_2-3X_0)X_0=0$

$AP_{2\infty}: \frac{x-1}{1}=\frac{y-1}{2}$ $\quad 2x-2=y-1$ $\quad 2x-y-1=0$ $\qquad AP_{1\infty}: \frac{x-1}{2}=\frac{y-1}{-1}$ $\quad -x+1=2y-2$ $\quad x+2y-3=0$

$f: -2y^2 + 3xy + 2ky + 4y + 2x^2 + kx - 13x - 3k + 6 = 0$

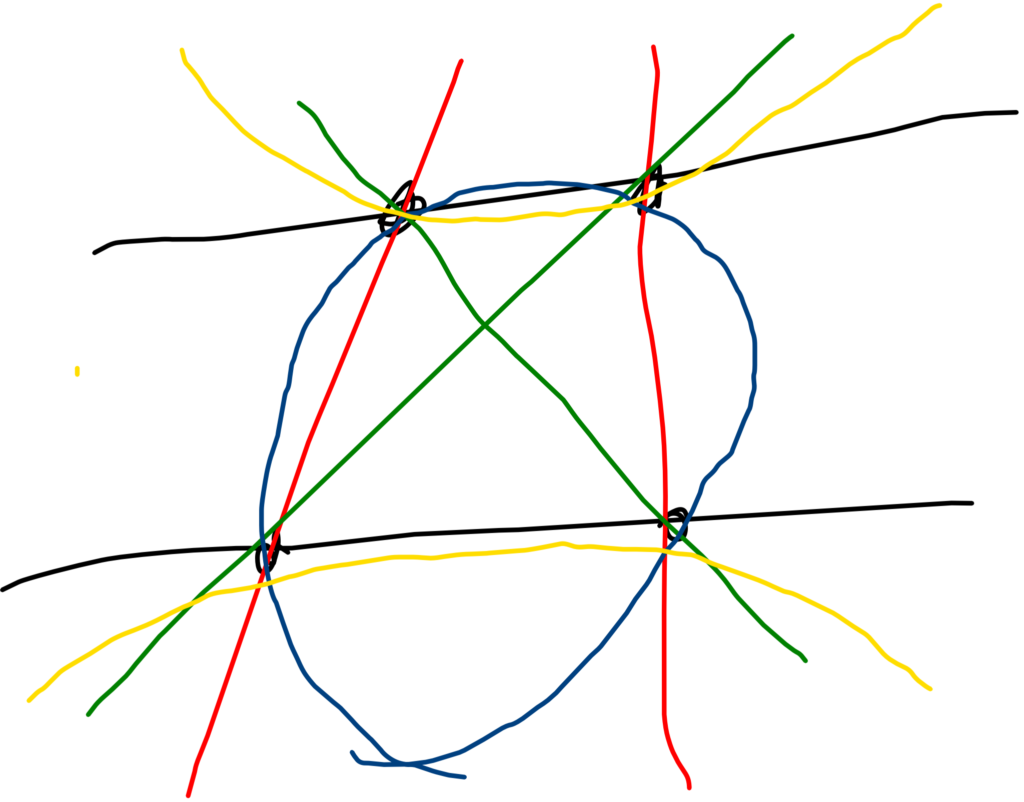
$$A = \begin{pmatrix} (6-3k) & \frac{(k-13)}{2} & (k+2) \\ \frac{(k-13)}{2} & 2 & \frac{3}{2} \\ (k+2) & \frac{3}{2} & -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$$

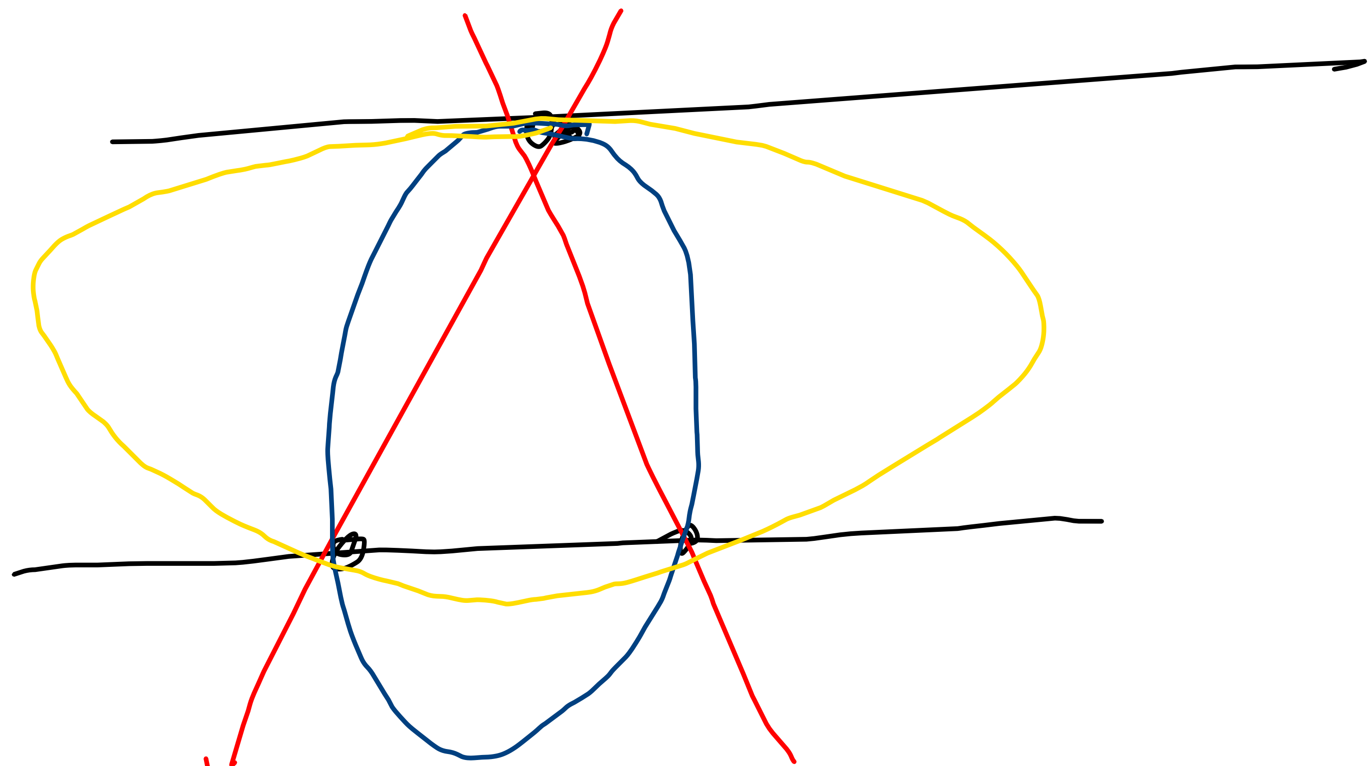
$(0\ 1\ 2) \cdot$

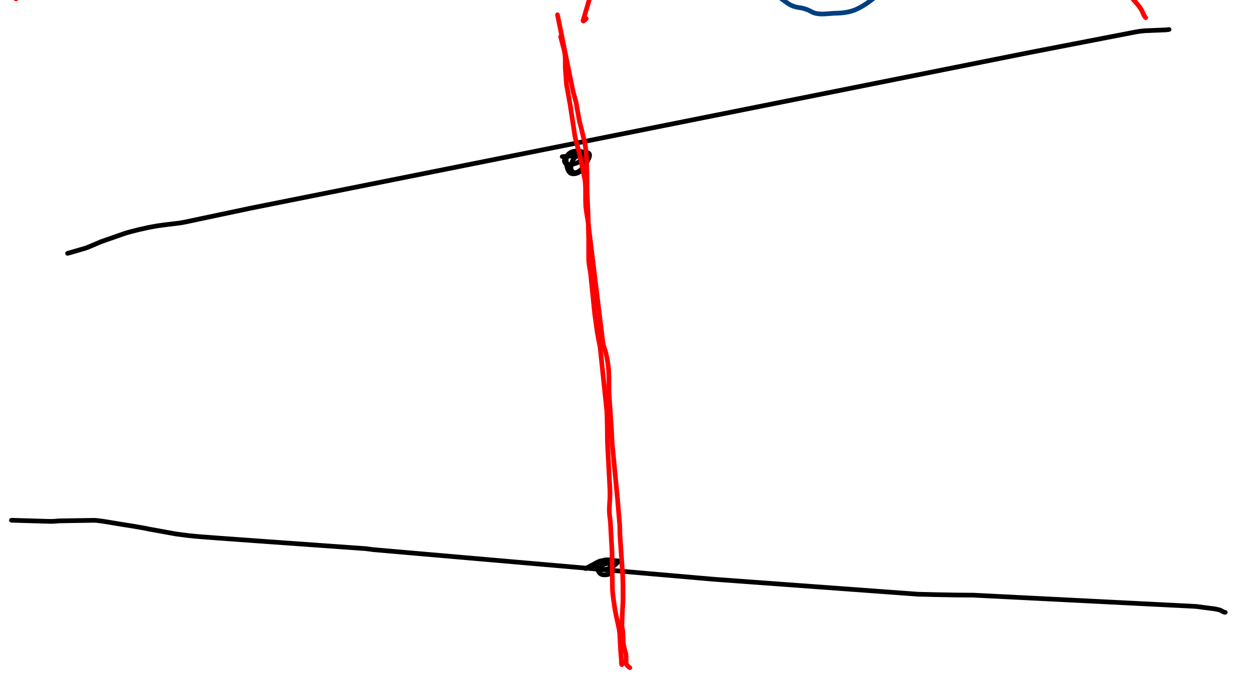
$$\left(\frac{k-13}{2} + 2k+4 \right) + 5x + \left(\frac{3}{2} - 4\right) y = 0$$

$$\frac{5k-9}{2} + 5x - \frac{5}{2} y = 0$$

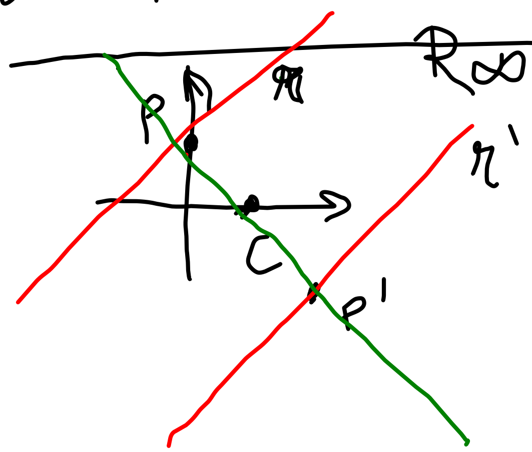
$$\cdot \frac{2}{5} \qquad \left(k - \frac{9}{5}\right) + 2x - y = 0$$







Esempio 4 es. 1 Fascio di coniche aventi centro $C \equiv (1,0)$, aventi come tang. in $P \equiv (0,1)$ la retta $r : x-y+1=0$.



$$\frac{P+P'}{2} = C \qquad P' = 2C - P \equiv (2,0) - (0,1) = (2,-1)$$

$$R_\infty \equiv (0,1,1)$$

$$r : x-y+1=0$$

$$r' : 1(x-2) - 1(y+1) = 0$$

$$x - y - 3 = 0$$

$$PP' : \frac{x-0}{2-0} = \frac{y-1}{-1-1} \qquad \frac{x}{2} = \frac{y-1}{-2}$$

$$-2x = 2y - 2$$

$$x + y - 1 = 0$$

$\Gamma_1 = r \cup r'$

$\Gamma_2 = PP'$ 2 volte

$$\Gamma_1 : (x-y+1)(x-y-3) = 0$$

$$\Gamma_2 : (x+y-1)^2 = 0$$

$$\mathcal{F} : \alpha(x-y+1)(x-y-3) + \beta(x+y-1)^2 = 0$$

$$\alpha(y^2 - 2xy + 2y + x^2 - 2x - 3) + \beta(x^2 + y^2 + 1 + 2xy - 2x - 2y) = 0$$

k

$$(1+k)x^2 + 2(k-1)xy + (1+k)y^2 - 2(1+k)x + 2(1-k)y + k - 3 = 0$$

$$(1+k)x^2+2(k-1)xy+(1+k)y^2-2(1+k)x+2(1-k)y+k-3=0$$

$$A_k=\begin{pmatrix} (k-3) & -(1+k) & (1-k) \\ -(1+k) & (1+k) & (k-1) \\ (1-k) & (k-1) & (1+k) \end{pmatrix}$$

M_3 M_2 M_1

$|A_k| = -16k$ deg $\Leftrightarrow k=0$

$A_0^0 = |M_0^0| = 4k$

ip. 0 ell.

$|M_1| = 1+k$

$|M_2| = 4k$

$|M_3| = -16k$

	$k<-1$	$-1<k<0$	$k>0$
$\lvert M_1\rvert$	$-$	$+$	$+$
$\lvert M_2\rvert$	$-$	$-$	$+$
$\lvert M_3\rvert$	$+$	$+$	$-$
	indef.	indef	indef.

$*$

k	$\lvert A_k\rvert$	$\lvert M_0^0\rvert$	coniche
$k<0$	$\neq 0$	$-$	iperboli
$k=0$	$=0$	0	deg. rg $=1$
$k>0$	$\neq 0$	$+$	ellissi reali $*$

Fra le coniche di $\mathcal{F}$ determinare l'iperbole avente un asintoto $\parallel$ asse x

$$\begin{cases} (1+k)x^2+2(k-1)xy+(1+k)y^2-2(1+k)xt+2(1-k)yt+(k-3)t^2=0 \\ t=0 \end{cases}$$

$$\begin{cases} (1+k)x^2+2(k-1)xy+(1+k)y^2=0 \\ t=0 \end{cases}$$

impongo che contenga

$(t, x, y) = (0, 1, 0) \equiv P_{1\infty}$

$(1+k)\cdot 1+2(k-1)\cdot 1\cdot 0+(1+k)\cdot 0^2=0$

$1+k=0 \qquad k=-1$

$-4xy+4y-4=0$

$$\begin{cases} -4xy=0 \\ t=0 \end{cases} \quad \begin{cases} x=0 \leftarrow P_{2\infty} \equiv (0,0,1) \\ y=0 \quad P_{1\infty} \end{cases}$$

$$= \begin{pmatrix} (k-3) & -(1+k) & (1-k) \\ -(1+k) & (1+k) & (k-1) \\ (1-k) & (k-1) & (1+k) \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} -4 & 0 & 2 \\ 0 & 0 & -2 \\ 2 & -2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ x \\ y \end{pmatrix} = 0$$

$$2 - 2x = 0$$