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GEOMETRICO-STATIC ANALYSIS OF UNDER-CONSTRAINED CABLE-DRIVEN PARALLEL ROBOTS

Marco Carricato

UNIVERSITY OF BOLOGNA

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2. Geometrico-Static Model
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1. Introduction

Fully-constrained Cable-Driven Parallel Robots (CDPRs):

- they have at least 6 cables *in tension*;
- the platform posture is completely determined by geometric constraints (i.e. cable lengths);
- they have been amply studied in the literature.

Under-constrained Cable-Driven Parallel Robots (CDPRs):

- they have less than 6 cables *in tension*;
- the platform posture is determined by geometric constraints and equilibrium equations;
- main challenges:
  - kinematics and statics/dynamics are coupled and must be solved simultaneously;
  - stability must be considered;
- they have been little studied in the literature;.

2. Geometrico-static model

Geometric constraints:

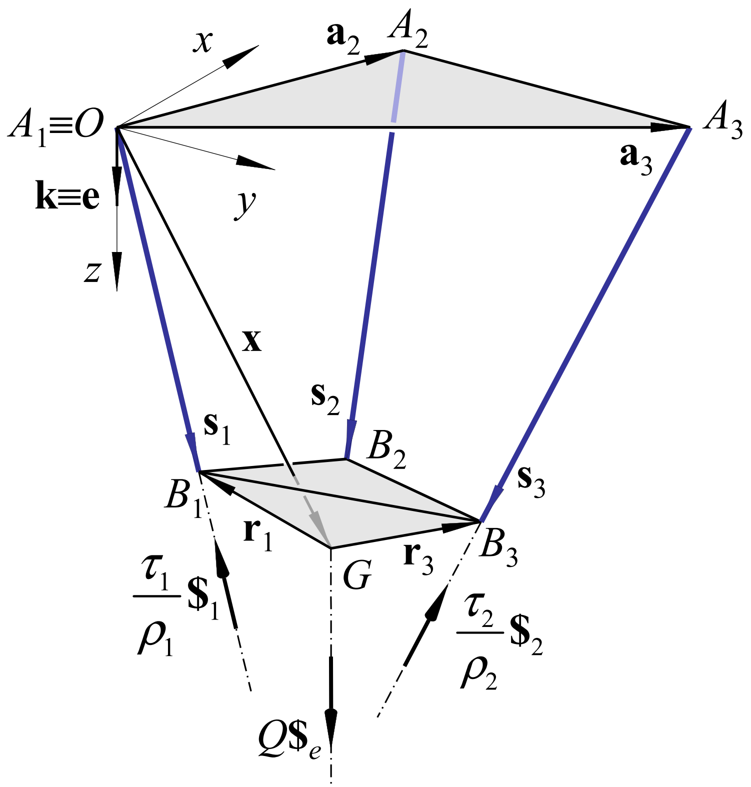
$$\mathbf{s}_i \cdot \mathbf{s}_i = \rho_i^2, \quad i = 1 \ldots n$$

Equilibrium constraints:

$$\begin{cases} \sum_{i=1}^{n} \frac{\tau_i}{\rho_i} \$_i + Q\$_e = \mathbf{0} \\ \tau_i \geq 0, \quad i = 1 \ldots n \end{cases}$$

Geometrico-static problem:

- $6+n$ scalar equations.
- $6+2n$ variables: $\mathbf{X} \equiv (\mathbf{x}; \mathbf{\Phi})$, (τ_i, ρ_i), $i = 1 \ldots n$.



$$\$_e = [\mathbf{e};\ \mathbf{x} \times \mathbf{e}], \ \$_i = [-\mathbf{s}_i;\ -\mathbf{a}_i \times \mathbf{s}_i]$$

Inverse geometrico-static problem (IGP):

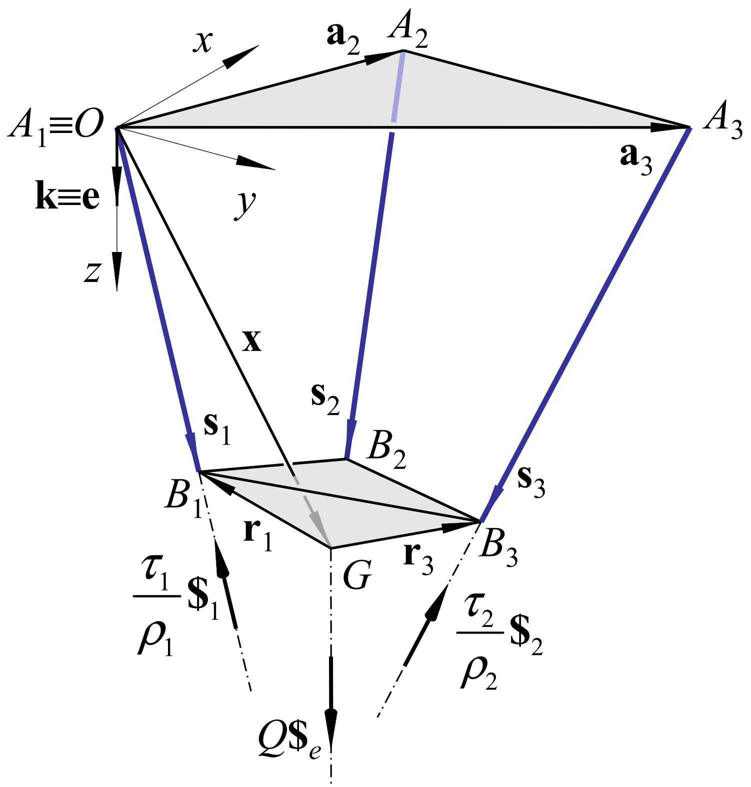
- n vars. in $\mathbf{X} \rightarrow 6-n$ vars. in $\mathbf{X}$, (τ_i, ρ_i), $i = 1 \ldots n$.

Direct geometrico-static problem (DGP):

- ρ_i, $i = 1 \ldots n \rightarrow \mathbf{X}$, τ_i, $i = 1 \ldots n$.

$\Downarrow$

6+n scalar equations 6+n variables



$$\mathbf{\$}_e = [\mathbf{e};\ \mathbf{x} \times \mathbf{e}],\ \mathbf{\$}_i = [-\mathbf{s}_i;\ -\mathbf{a}_i \times \mathbf{s}_i]$$

Direct elimination of cable tensions from equilibrium constraints:

$$6\text{ eqs.}\left\{\sum_{i=1}^{n}\frac{\tau_i}{\rho_i}\mathbf{\$}_i + Q\mathbf{\$}_e = \mathbf{0}\right. \quad\Rightarrow\quad \frac{\tau_i}{\rho_i} = \frac{\tau_i}{\rho_i}(\mathbf{X}),\ i = 1\ldots n \quad\Rightarrow\quad 6-n \text{ relations in } \mathbf{X}$$

Alternative strategy to eliminate cable tensions from equilibrium constraints:

$\mathbf{\$}_1, \ldots, \mathbf{\$}_n, \mathbf{\$}_e$ must be linearly dependent

$$\Rightarrow\ \mathbf{M} = \underbrace{\begin{bmatrix}\mathbf{\$}_1 & \cdots & \mathbf{\$}_n & \mathbf{\$}_e\end{bmatrix}}_{6\times(n+1)} \ \Rightarrow\ \operatorname{rank}(\mathbf{M}) \le n \ \Rightarrow\ \text{all } (n+1)\times(n+1) \text{ minors must} = 0$$

$$\Rightarrow\ \text{scalar relations}: p_j(\mathbf{X}) = 0,\ j = 1\ldots C_{n+1}^{6},\quad \text{with } C_{n+1}^{6} = \binom{6}{n+1}$$

Advantages:

- more than $6-n$ relations in $\mathbf{X}$ are obtained;
- better insight into equation structure;
- partial decoupling of system equations.

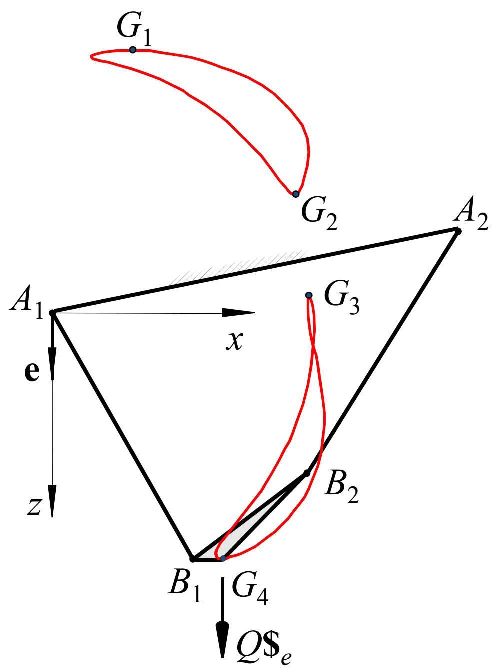
Partially-decoupled IGP:

- $p_j(\mathbf{X}) = 0 \rightarrow 6-n$ unknowns in $\mathbf{X}$;
- $\rho_i = |\mathbf{s}_i| \rightarrow \rho_i,\ i = 1\ldots n$.

3. Stability of equilibrium

Physical interpretation:

- the constrained platform has $6-n$ dofs;
- it is in equilibrium in all points in which the potential energy $U=-Q\mathbf{e}\cdot\mathbf{x}$ is stationary;
- the equilibrium is stable if the stationary point of U is a minimum.



Unconstrained-potential approach (Michael *et al.* 2009):

$$\mathbf{X} = [\underbrace{\mathbf{X}_{ind}}_{6-n}; \underbrace{\mathbf{X}_{dep}}_{n}] \Rightarrow \mathbf{X}_{dep} = \mathbf{X}_{dep}(\mathbf{X}_{ind}) \Rightarrow U(\mathbf{X}_{ind})$$

$$\begin{cases} \nabla U(\mathbf{X}_{ind}) = \mathbf{0} \Rightarrow \text{Stationary configurations (Equilibrium configurations)} \\ \mathbf{H}_U(\mathbf{X}_{ind}) \Rightarrow \text{Minima / Maxima / Saddles (Stability)} \end{cases}$$

- it is (very) difficult to obtain $\mathbf{X}_{dep} = \mathbf{X}_{dep}(\mathbf{X}_{ind})$;
- it requires extensive symbolic computation (for implicit differentiation) and geometry simplifications.

Constrained-potential approach:

- Lagrange function: $L(\mathbf{X},\tau_i) = -Q\mathbf{e}\cdot\mathbf{x} + \sum_{i=1}^{n}\tau_i\left(|\mathbf{s}_i| - \rho_i\right)$
- Stability may be assessed by the Hessian $\mathbf{H}_r$ of L restricted to the tangent space of the constraint equations, namely:

$$\delta^2 L = \delta\mathbf{t}^T\mathbf{H}(\mathbf{X})\delta\mathbf{t} \;\Rightarrow\; \mathbf{H}(\mathbf{X}) = \sum_{i=1}^{n}\frac{\tau_i}{\rho_i}\begin{bmatrix}\mathbf{I}_3 & -\tilde{\mathbf{r}}_i \\ \tilde{\mathbf{r}}_i & \frac{1}{2}\left(\tilde{\mathbf{r}}_i\tilde{\mathbf{x}} - \tilde{\mathbf{r}}_i\tilde{\mathbf{a}}_i + \tilde{\mathbf{x}}\tilde{\mathbf{r}}_i - \tilde{\mathbf{a}}_i\tilde{\mathbf{r}}_i\right)\end{bmatrix}$$

$$\Downarrow$$

$$\mathbf{J} = \begin{bmatrix}\mathbf{s}_1^T & (\mathbf{r}_1\times\mathbf{s}_1)^T \\ \vdots & \vdots \\ \mathbf{s}_n^T & (\mathbf{r}_n\times\mathbf{s}_n)^T\end{bmatrix} \;\Rightarrow\; \mathbf{N} = \ker(\mathbf{J}) \;\Rightarrow\; \boxed{\mathbf{H}_r = \mathbf{N}^T\mathbf{H}\,\mathbf{N}}$$

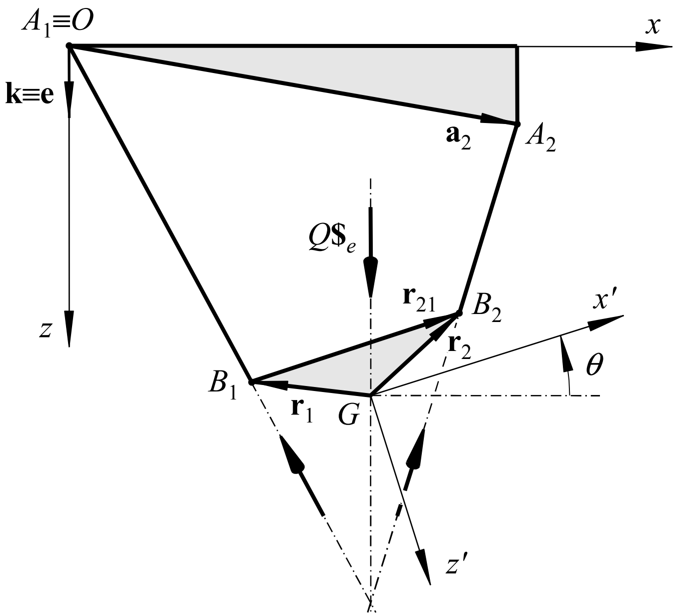
Advantages:

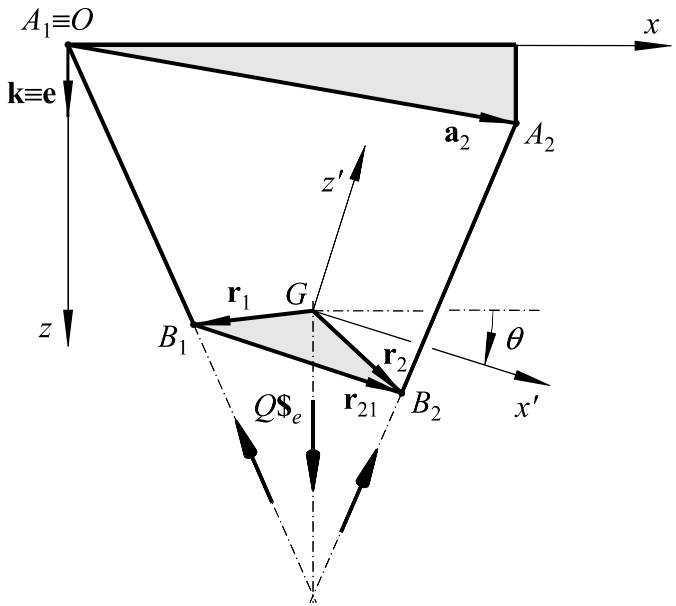
- simple linear-algebra computation (no symbolic calculation, no differentiation),
- straightforward application to any robot geometry.

4. 22-CDPR

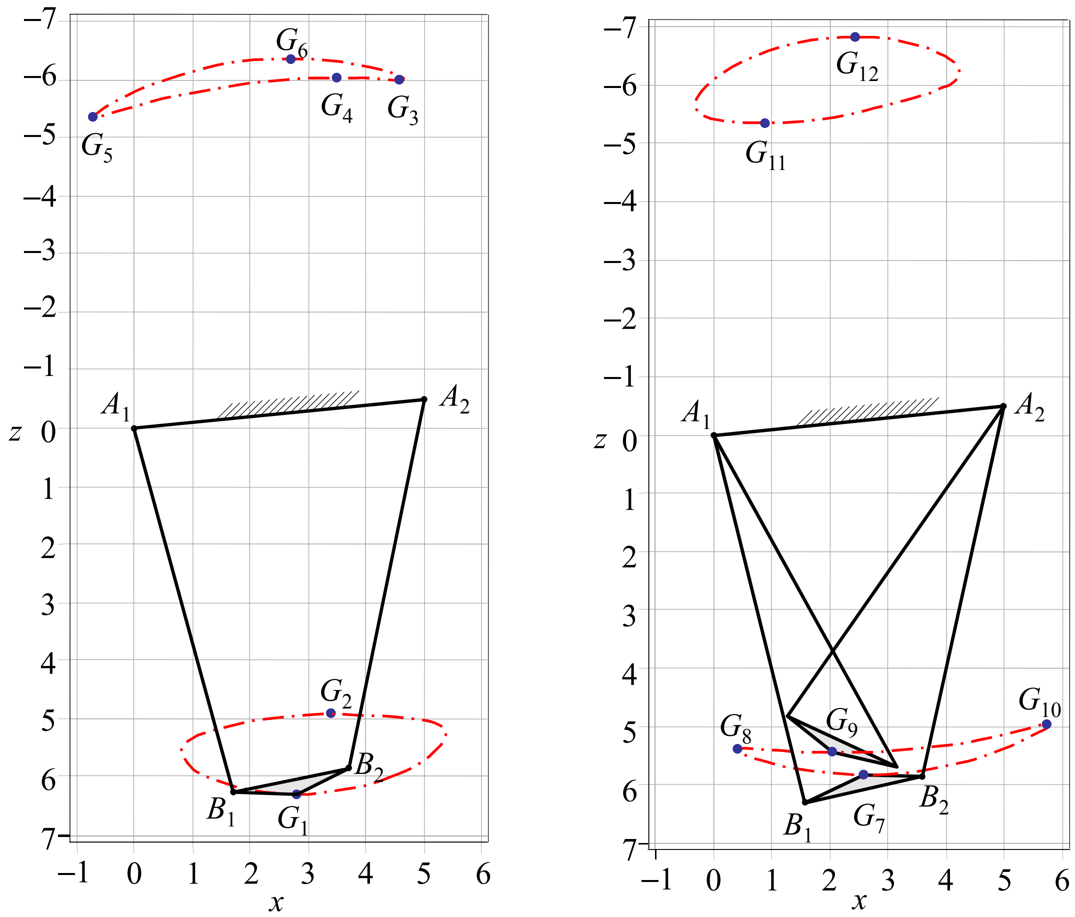
DGP:

- the generic coupler curve *g* of a 4-bar linkage is a sextic of class 12 (Hunt 1978);
- hence, there are up to 12 lines tangent to *g* and perpendicular to **e**, and 12 (complex) solutions for the DGP of the 22-CDPR;
- however, the 22-CDPR has 2 *operation modes*:



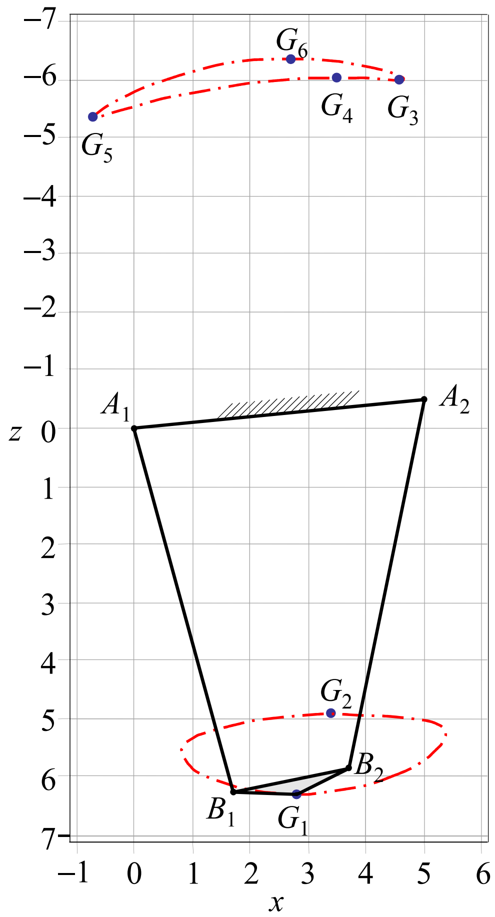


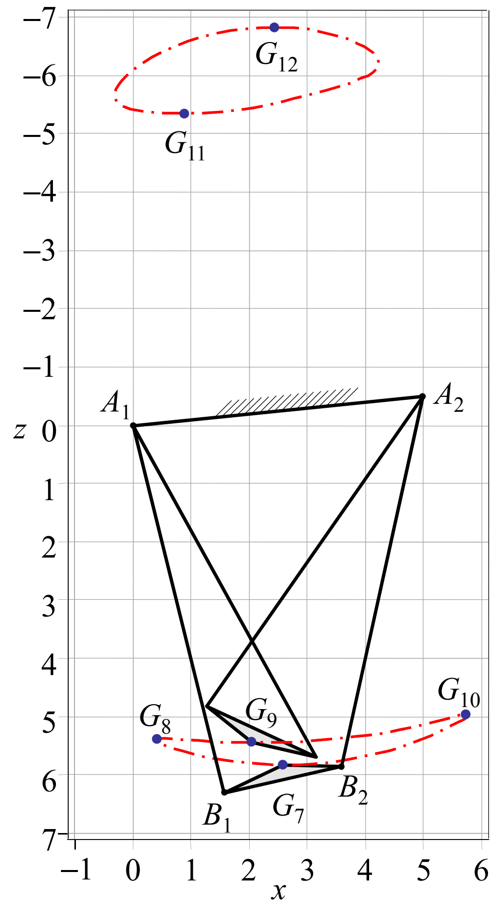
- the DGP of the 22-CDPR admits 12 distinct solutions *per mode*, plus potential solutions with only one cable in tension:



- G_1, G_7 and G_9 are the stable feasible configurations if only *planar* movements are considered;
- G_1 is the only stable feasible configuration if general *spatial* movements are considered.

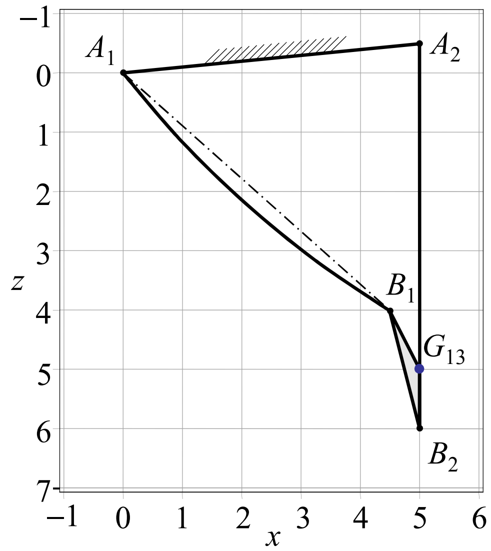
- the DGP of the 22-CDPR admits 12 distinct solutions *per mode*, plus potential solutions with only one cable in tension:





Actual constraints:

$$|\mathbf{s}_i| \le \rho_i, \quad i = 1 \ldots n$$



$$\{|\mathbf{s}_2| = \rho_2, \ |\mathbf{s}_1| < \rho_1\}$$

5. 33-CDPR

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6. Conclusions

This paper presented a general method for the geometrico-static analysis of CDPRs, and in particular:

- stability analysis by a simple linear-algebra-based constrained-optimization approach;
- effective strategy for the solution of the inverse and direct position problems in analytical form.

Application examples:

- 22-CDPR:
  - DGP: 24 solutions.
- 33-CDPR:
  - IGP with orientation assigned: 1 solution;
  - IGP with position assigned: 24 solutions;
  - DGP: 156 solutions.

Reference:

Carricato, M., and Merlet, J.-P. 2013.
Stability Analysis of Underconstrained Cable-Driven Parallel Robots.
IEEE Transactions on Robotics, 29(1), pp. 288-296.

Thank you very much for your attention!

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

UNIONE EUROPEA
Fondo europeo di sviluppo regionale
POR FESR 2007-2013
OBIETTIVO COMPETITIVITÀ
E OCCUPAZIONE
Regione Emilia-Romagna
COSTRUIAMO INSIEME IL FUTURO

RETE ALTA TECNOLOGIA
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