$$
\begin{tikzcd}
0 \arrow[r] & C'_{k+1} \arrow[r, "\alpha"] \arrow[d, "\partial'"] & C_{k+1} \arrow[r, "\beta"] \arrow[d, "\partial"] & C''_{k+1} \arrow[r] \arrow[d, "\partial''"] & 0 \\
0 \arrow[r] & C'_{k} \arrow[r, "\alpha"] \arrow[d, "\partial'"] & C_{k} \arrow[r, "\beta"] \arrow[d, "\partial"] & C''_{k} \arrow[r] \arrow[d, "\partial''"] & 0 \\
0 \arrow[r] & C'_{k-1} \arrow[r, "\alpha"] \arrow[d, "\partial'"] & C_{k-1} \arrow[r, "\beta"] \arrow[d, "\partial"] & C''_{k-1} \arrow[r] & 0 \\
0 \arrow[r] & C'_{k-2} \arrow[r, "\alpha"] & C_{k-2} & &
\end{tikzcd}
$$

$$
\begin{tikzcd}
d \arrow[r, mapsto] \arrow[d, mapsto] & d'' \arrow[d, mapsto] & \\
C_{k+1} \arrow[r, "\beta"] \arrow[d, "\partial"] & C''_{k+1} \arrow[r] \arrow[d, "\partial''"] & 0 \\
\partial d \arrow[r, mapsto] & \partial'' d'' & \\
c_1 \arrow[r, mapsto] & z'' + \partial'' d'' & \\
C_k \arrow[r, "\beta"] & C''_k \arrow[r] & 0 \\
c \arrow[r, mapsto] & z'' &
\end{tikzcd}
$$

$$\beta(c_1) = z'' + \partial'' d'' = \beta(c) + \beta(\partial d) = \beta(c + \partial d) \Rightarrow \beta(c_1 - c - \partial d) = 0$$

$$
\begin{tikzcd}
0 \arrow[r] & C'_k \arrow[r, "\alpha"] \arrow[d, "\partial'"] & C_k \arrow[r, "\beta"] \arrow[d, "\partial"] & C''_k \arrow[r] & 0 \\
& d' \arrow[r, mapsto] & c_1 - c - \partial d \arrow[r, mapsto] \arrow[d, mapsto] & 0 & \\
& C'_{k-1} \arrow[r] & C_{k-1} \quad \partial c_1 - \partial c - \cancel{\partial\partial} d & &
\end{tikzcd}
$$

$c_1 - c - \partial d = \alpha d'$

$c_1 = c + \partial d + \alpha d'$ $\quad \partial c_1 = \partial c + \cancel{\partial\partial} d + \partial \alpha d' = \alpha c' + \alpha \partial' d' =$
$= \alpha(c' + \partial' d')$

perciò $\alpha^{-1}(\partial c_1) = c' + \partial' d'$

E' ben definita la classe $\{\alpha^{-1}\partial\beta^{-1}z''\} \in H_{k-1}(C')$ a partire $\{z''\} \in H_k(C'')$

$$\begin{tikzcd}
d' \arrow[r, mapsto] & \alpha d' & c \arrow[r, mapsto] & z'' & \\
C'_k \arrow[r, "\alpha"] \arrow[d, "\partial'"] & C_k \arrow[r, "\beta"] \arrow[d, "\partial"] & C''_k \arrow[r] & 0 \\
C'_{k-1} \arrow[r, "\alpha"] & C_{k-1} & & \\
\alpha^{-1}\partial c \arrow[r, mapsto] & \partial c & &
\end{tikzcd}$$

$$\partial(c - \alpha d') = 0$$

$$
\begin{tikzcd}
G_5 \arrow[r, "\alpha_5"] \arrow[d, two heads, "\gamma_5"] & G_4 \arrow[r, "\alpha_4"] \arrow[d, tail, "\gamma_4"] & G_3 \arrow[r, "\alpha_3"] \arrow[d, "\gamma_3"] & G_2 \arrow[r, "\alpha_2"] \arrow[d, tail, "\gamma_2"] & G_1 \arrow[d, "\gamma_1"] \\
H_5 \arrow[r, "\beta_5"] & H_4 \arrow[r, "\beta_4"] & H_3 \arrow[r, "\beta_3"] & H_2 \arrow[r, "\beta_2"] & H_1
\end{tikzcd}
$$

$$
\begin{tikzcd}
g_5 \arrow[r, mapsto] \arrow[d, mapsto] & \alpha_5 g_5 = g_4 \arrow[r, mapsto] \arrow[d, mapsto] & 0 = g_3 \arrow[r, mapsto] \arrow[d, mapsto] & 0 \arrow[d, mapsto] \\
h_5 \arrow[r, mapsto] & h_4 \arrow[r, mapsto] & 0 \arrow[r, mapsto] & 0
\end{tikzcd}
$$

$$
\begin{tikzcd}
G_5 \arrow[r, "\alpha_5"] \arrow[d, "\gamma_5"] & G_4 \arrow[r, "\alpha_4"] \arrow[d, "\gamma_4"] & G_3 \arrow[r, "\alpha_3"] \arrow[d, "\gamma_3"] & G_2 \arrow[r, "\alpha_2"] \arrow[d, "\gamma_2"] & G_1 \arrow[d, "\gamma_1"] \\
H_5 \arrow[r, "\beta_5"] & H_4 \arrow[r, "\beta_4"] & H_3 \arrow[r, "\beta_3"] & H_2 \arrow[r, "\beta_2"] & H_1
\end{tikzcd}
$$

$g_4 \mapsto \alpha_4 g_4$

$g_3 \mapsto g_2 \mapsto 0$

$h_3 \mapsto h_2 \mapsto 0$

$h_4 \mapsto -\gamma_3 g_3 + h_3$

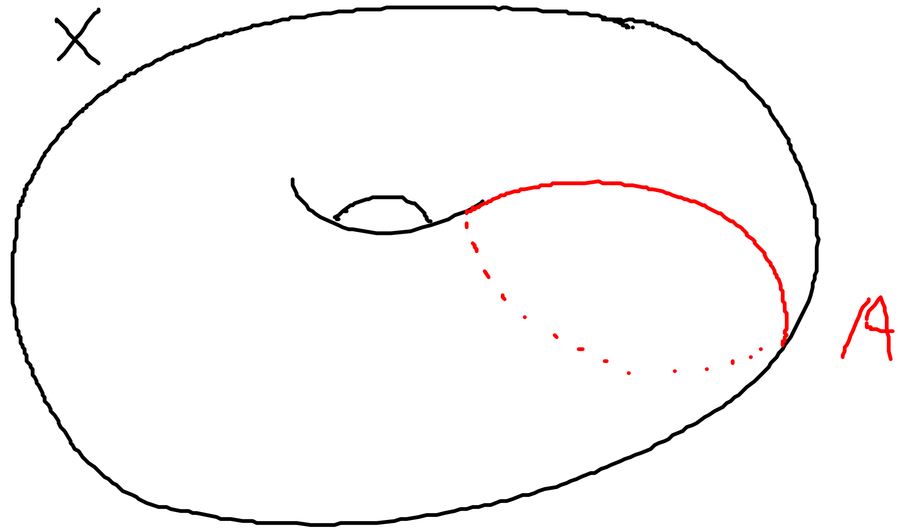
$-\gamma_3 g_3 \mapsto -h_2$

$$\gamma_3(g_3 + \alpha_4 g_4) = \cancel{\gamma_3 g_3} + h_3 - \cancel{\gamma_3 g_3} = h_3$$

$$\begin{tikzcd}
0 \arrow[r] & S(A) \arrow[r, "i"] \arrow[d, "f"] & S(X) \arrow[r, "j"] \arrow[d, "f"] & S(X,A) \arrow[r] \arrow[d, "f"] & 0 \\
0 \arrow[r] & S(B) \arrow[r] & S(Y) \arrow[r] & S(Y,B) \arrow[r] & 0
\end{tikzcd}$$

$$H_k(X,A) \cong \tilde{H}_k(X/A)$$

quando esiste in X un intorno di A di cui A sia retratto per deformazione


X
A