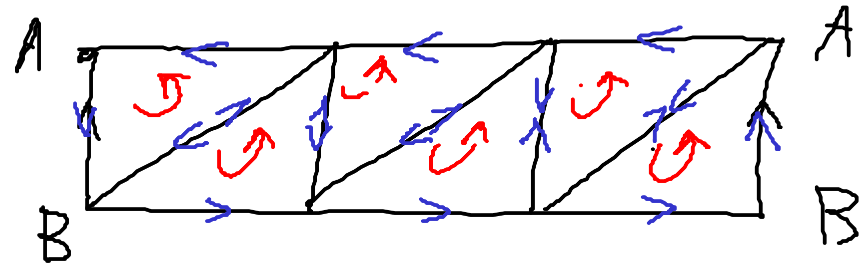
A
A
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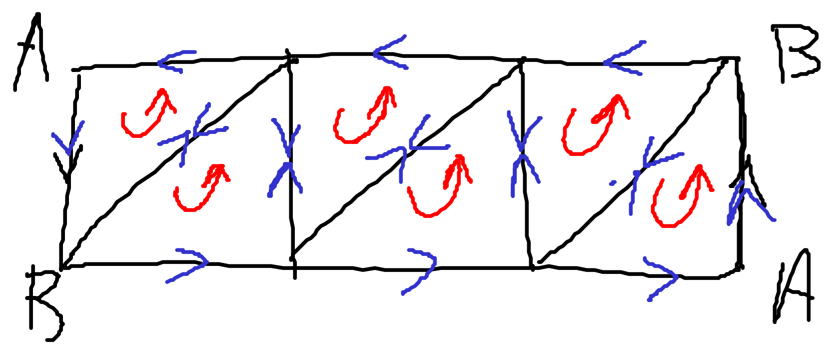
A
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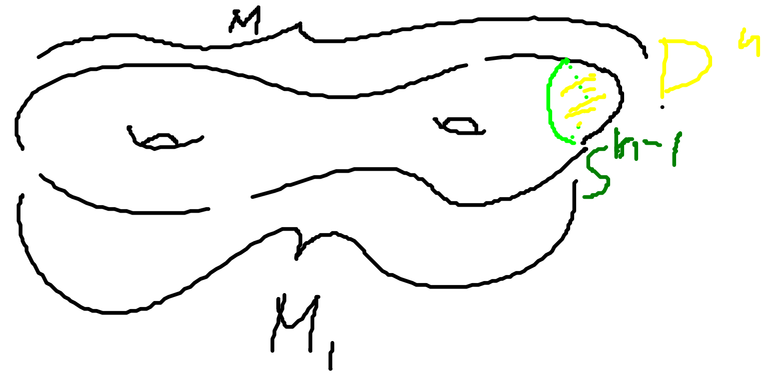

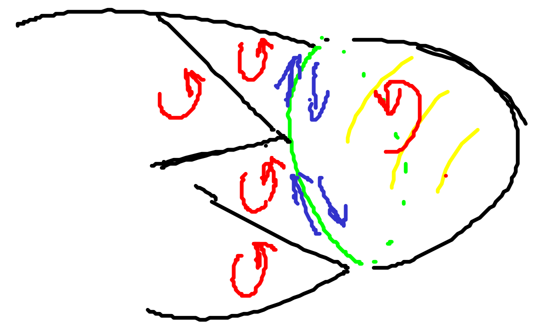

$$H_n(M_1)+H_n(D^n) \to H_n(M) \longrightarrow H_{n-1}(S^{n-1}) \longrightarrow H_{n-1}(M_1)\oplus H_{n-1}(D^n) \longrightarrow H_{n-1}(M) \longrightarrow H_{n-2}(S^{n-1})$$

$$0 \quad 0 \qquad \mathbb{Z} \xrightarrow{\cong} \mathbb{Z} \longmapsto 0 \qquad\qquad 0 \quad \cong \qquad \qquad 0$$

cl. fond. di M

(z_1, z_2) $\qquad\qquad$ $z \longmapsto c$

$$C_n(M_1)\oplus C_n(D^n) \longrightarrow C_n(M) \longrightarrow 0$$

$$\downarrow \partial\oplus\partial \qquad\qquad \downarrow$$

$$C_{n-1}(S^{n-1}) \longrightarrow C_{n-1}(M_1)\oplus C_{n-1}(D^n) \qquad 0$$

$\partial z_1 \longmapsto (\partial z_1, \partial z_2)$, $\partial z_2 = -\partial z_1$

cl. fond. di S^{n-1}

somme degli $(n-1)$-simpl. di S^{n-1}, con segni opposti

$$H_n(M_1)\oplus H_n(N_1) \to H_n(M\# N) \to H_{n-1}(S^{n-1}) \to H_{n-1}(M_1)\oplus H_{n-1}(N_1) \to H_{n-1}(M\# N) \to H_{n-2}(S^{n-1})$$

$$0 \qquad 0 \qquad \mathbb{Z} \xrightarrow{\cong} \mathbb{Z} \to 0 \subset \qquad \cong \qquad 0$$

$$(z_M, z_N) \longmapsto z \quad \text{cl. fund. di } M\# N$$

$$\mathcal{C}_n(M_1)\oplus \mathcal{C}_n(N_1) \longrightarrow \mathcal{C}_n(M\# N)$$

$$\downarrow \partial\oplus\partial$$

$$\mathcal{C}_{n-1}(S^{n-1}) \longrightarrow \mathcal{C}_{n-1}(M_1)\oplus \mathcal{C}_{n-1}(N_1)$$

cl. fund. di S^{n-1} $\quad \partial z_M \longmapsto (\partial z_M, \partial z_N)$, $\quad \partial z_N = -\partial z_M$

$$H_{n-1}(M\# N) \cong H_{n-1}(M_1)\oplus H_{n-1}(N_1) \cong H_{n-1}(M)\oplus H_{n-1}(N)$$