alessia.cattabriga@unibo.it

L6

$$p : \mathbb{R} \longrightarrow S^1 \subseteq \mathbb{C}$$

$$x \longmapsto e^{ix}$$

$$\cancel{p^{-1}(0)} \qquad p^{-1}(1) = \{2k\pi : k \in \mathbb{Z}\}$$

$$p : \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R} \times S^1$$

$$(x, y) \longrightarrow (x, e^{iy})$$

BUON COMPORT. RISPETTO $\times$

$$\begin{array}{l} p_1 : \tilde{X}_1 \longrightarrow X_1 \\ p_2 : \tilde{X}_2 \longrightarrow X_2 \end{array} \Longrightarrow p_1 \times p_2 : \tilde{X}_1 \times \tilde{X}_2 \longrightarrow X_1 \times X_2$$

rivestiment. rivestimento

---

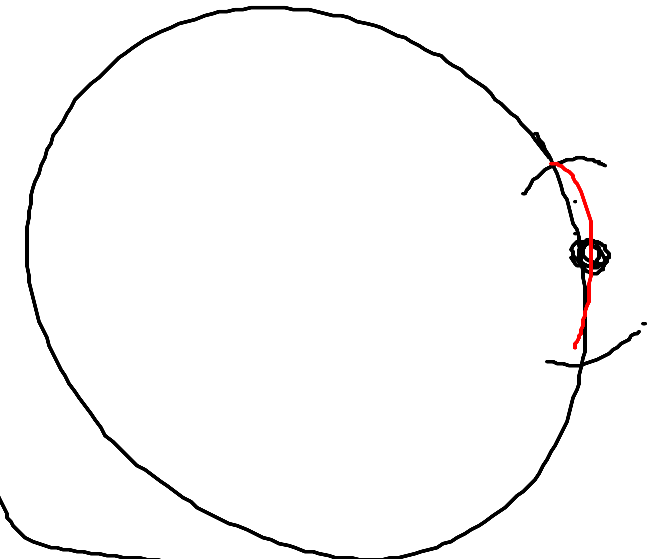
NON BUON COMPORT RESTIZIONE

$$p : \mathbb{R} \longrightarrow S^1$$

non e' un rivestimento $\longleftarrow$ $p|_{\mathbb{R}^+} : \mathbb{R}^+ \longrightarrow S^1$

$$\mathbb{R}\times\mathbb{R} \longrightarrow S^1 \times S^1$$

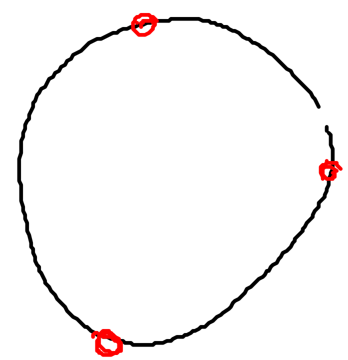
$$(x,y) \longmapsto (e^{ix}, e^{iy})$$

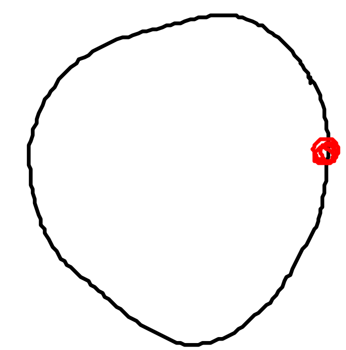


$$p_n : S^1 \longrightarrow S^1$$

$$z \longrightarrow z^n$$

$$|p_n^{-1}(z_0)| = n$$





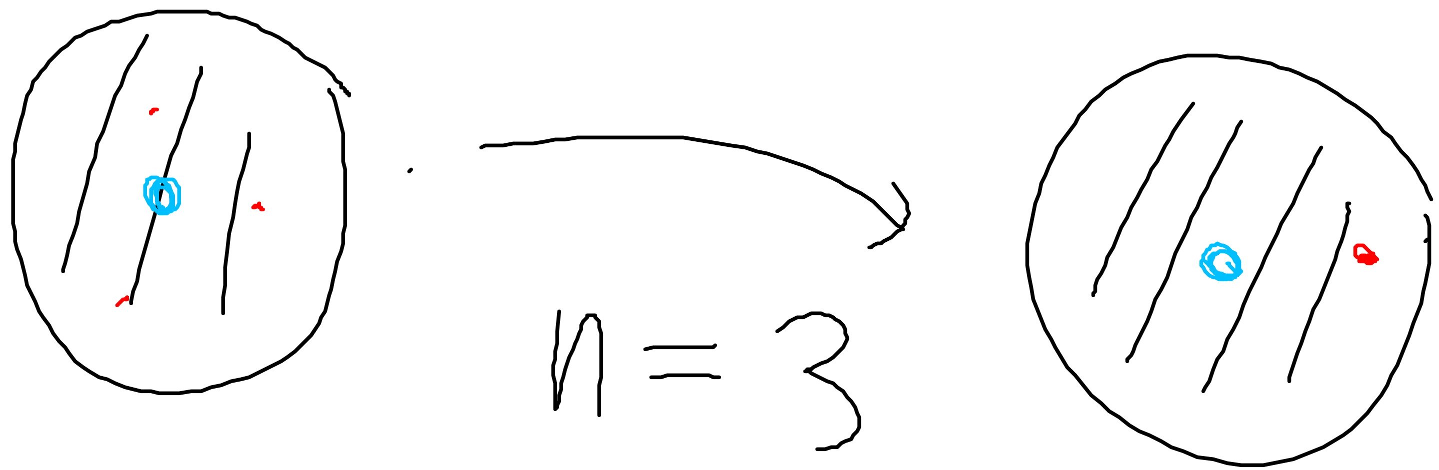
$n=3$

$$p : D^2 \longrightarrow D^2 = \{ z \in \mathbb{C} \mid |z| \leq 1 \}$$

$$z \longmapsto z^n$$

non è un rivestimento (rivestimento RAMIFICATO)

SE $n \geq 2$



$n = 3$

$$P^{-1}(z) \cong S^1$$

NON È DISCRETO $\Longrightarrow$ NON RIVESTIMENTO

$$P : \mathbb{C}^{n+1} \setminus \{\underline{0}\} \longrightarrow \mathbb{P}\mathbb{C}^n$$

$$z \longrightarrow [z] = \{\lambda z : \lambda \in \mathbb{C}^*\}$$

$$|z|^2 = x_0^2 + y_0^2 + x_1^2 + y_1^2 + \dots + x_n^2 + y_n^2 = 1$$

$$P : S^{2n+1} \longrightarrow \mathbb{P}\mathbb{C}^n$$

$$z \longrightarrow [z] = \{\lambda z : |\lambda| = 1\}$$

FIBRAZIONE

FIBRAZIONE

di

HOPF

$n = 1$

$$S^3 \longrightarrow \mathbb{CP}^1 \cong S^2$$

FIBRE SONO $S^1$

$$\mathbb{R}^{n+1} \supseteq S^n \xrightarrow{p} \mathbb{RP}^n$$

$$x \longmapsto [x]$$

$$p^{-1}(x) = \{\pm x\}$$

RIVESTIMENTO

E LE

FIBRE HANNO

CARDINALITÀ

2

· PROBLEMA del SOLLEVAMENTO

esiste $f'$?

unica?

$$\begin{tikzcd}
(\tilde{X}, \tilde{x}_0) \arrow[d, "p"'] & \\
(X, x_0) \arrow[r, phantom, "="] & (Y, y_0) \arrow[l, "f"] \arrow[ul, "f'"']
\end{tikzcd}$$

$f = p f'$

$Y$ connesso

Due sollevamenti di $f$ che
coincidono in un punto
coincidono sulla componente
connessa che contiene il punto.

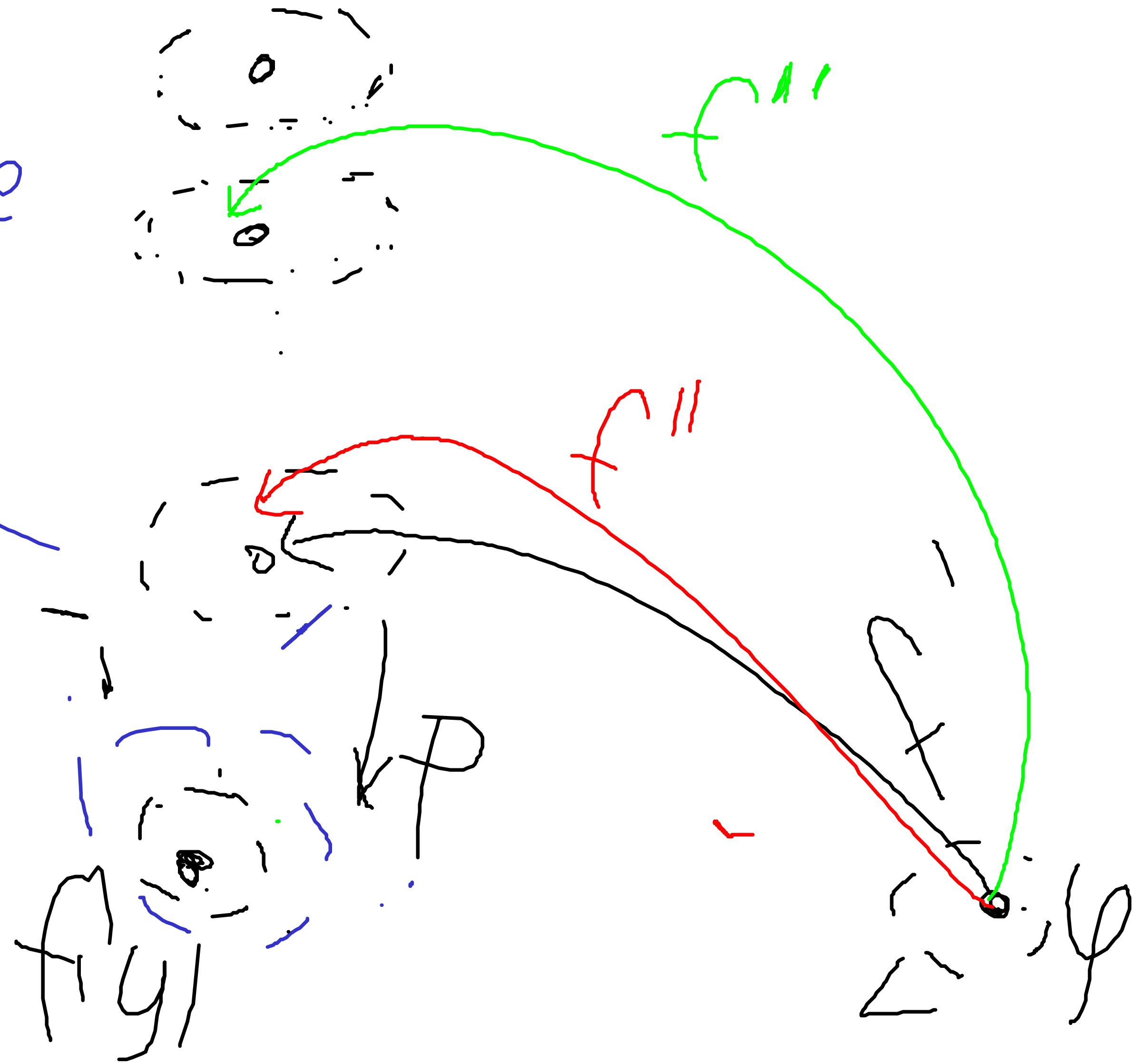
$$
\begin{tikzcd}
\tilde{x}_0 \quad \tilde{X} \arrow[d, "p"'] & \\
X \arrow[r, phantom, "\equiv f'"] & Y \arrow[l, "f"] \arrow[ul, bend right=20, "f'"'] \arrow[ul, bend right=40, "f''"'] \\
x_0 & y_0
\end{tikzcd}
$$

$y_0 \in A = \{ y \in Y : f'(y) = f''(y) \}$

$B = A^c \quad \neq$

U trivializzante

$P^{-1}(f(V))$ contenuto in una sola placca




$y \in Y$

$y \in A$

$y \in A^c$

f continua $\exists\, V \ni y$ intorno aperto $f(V) \subseteq U$

Se $y \in A^c \implies f'(V)$ e $f''(V)$ stanno in placche $\neq \implies f'(x) \neq f''(x) \ \forall x \in V \implies V \subseteq A^c \implies A^c$ aperto

Se $y \in A \Longrightarrow f'(V), f''(V)$

stanno nella stessa placca $\tilde{U}_1$

$$f'|_V = \left(p|_{\tilde{U}_1}\right)^{-1} f|_V = f''|_V \Longrightarrow V \subseteq A \Longrightarrow A \text{ aperto}$$

e $Y$ connesso $\Longrightarrow Y = A$ ∎