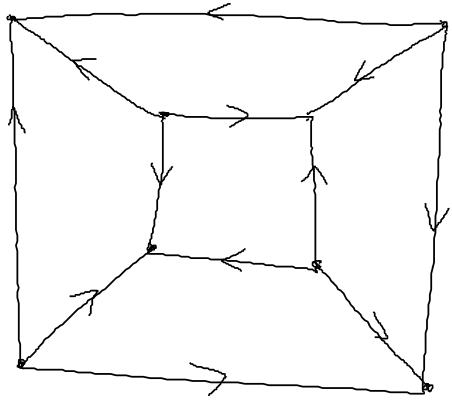


$$0 \rightarrow \mathbb{Z} \xrightarrow{1} \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{\pi} \mathbb{Z}_2 \rightarrow 0$$

$$1 \mapsto [1]_2$$

$$[0]_2$$

$$\otimes \mathbb{Z}$$



~~$$0 \rightarrow \mathbb{Z} \xrightarrow{[1]_2} \mathbb{Z} \xrightarrow{[2]_2} \mathbb{Z} \xrightarrow{1} \mathbb{Z} \rightarrow 0$$

$$[1]_2 \mapsto [1]_2$$~~

$$0 \rightarrow 0 \rightarrow \mathbb{Z} \xrightarrow{1} \mathbb{Z} \rightarrow 0$$

$$\otimes \mathbb{Z}$$

$$0 \rightarrow 0 \rightarrow \mathbb{Z} \rightarrow \mathbb{Z} \rightarrow 0$$

$$\text{Tor}(\mathbb{Z}, \mathbb{Z}) = 0$$

$$0 \rightarrow 0 \rightarrow \mathbb{Z}_2 \xrightarrow{1} \mathbb{Z}_2 \rightarrow 0$$

$$\otimes \mathbb{Z}_2$$

$$\text{Tor}(\mathbb{Z}_2, \mathbb{Z}_2) = 0$$

$$0 \rightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{\pi} \mathbb{Z}_2 \rightarrow 0$$

$$\otimes \mathbb{Z}$$

$$0 \rightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{\pi} \mathbb{Z}_2 \rightarrow 0$$

$$\text{Tor}(\mathbb{Z}_2, \mathbb{Z}) = 0$$

$$0 \rightarrow \mathbb{Z} \xrightarrow{6} \mathbb{Z} \xrightarrow{\pi} \mathbb{Z}_6 \rightarrow 0$$

$$\otimes \mathbb{Z}_{15}$$

$$\begin{matrix} [0]_{15} & [3]_{15} \\ [6]_{15} & [9]_{15} \\ [12]_{15} & \end{matrix}$$

$$0 \rightarrow \mathbb{Z}_3 \xrightarrow{[3]_3} \mathbb{Z}_{15} \xrightarrow{[m]_{15}} \mathbb{Z}_{15} \rightarrow \mathbb{Z}_3 \rightarrow 0$$

$$\begin{matrix} [0]_{15} \\ [5]_{15} \\ [10]_{15} \end{matrix} \mapsto [0]_{15}$$

$$[n]_{15} \mapsto [n]_3$$

$$\text{Tor}(\mathbb{Z}_{15}, \mathbb{Z}_6) \cong \mathbb{Z}_3$$

$$\text{Tor}(\mathbb{Z}_p, \mathbb{Z}_q) \cong \mathbb{Z}_{\text{gcd}(p,q)}$$

$$A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D \xrightarrow{\mathcal{D}} E$$

$$0 \longrightarrow \frac{B}{\text{Ker } g} \longrightarrow C \longrightarrow \text{Im } h \longrightarrow 0$$

$$0 \longrightarrow \text{Coker } f \longrightarrow C \longrightarrow \text{Ker } \mathcal{D} \longrightarrow 0$$