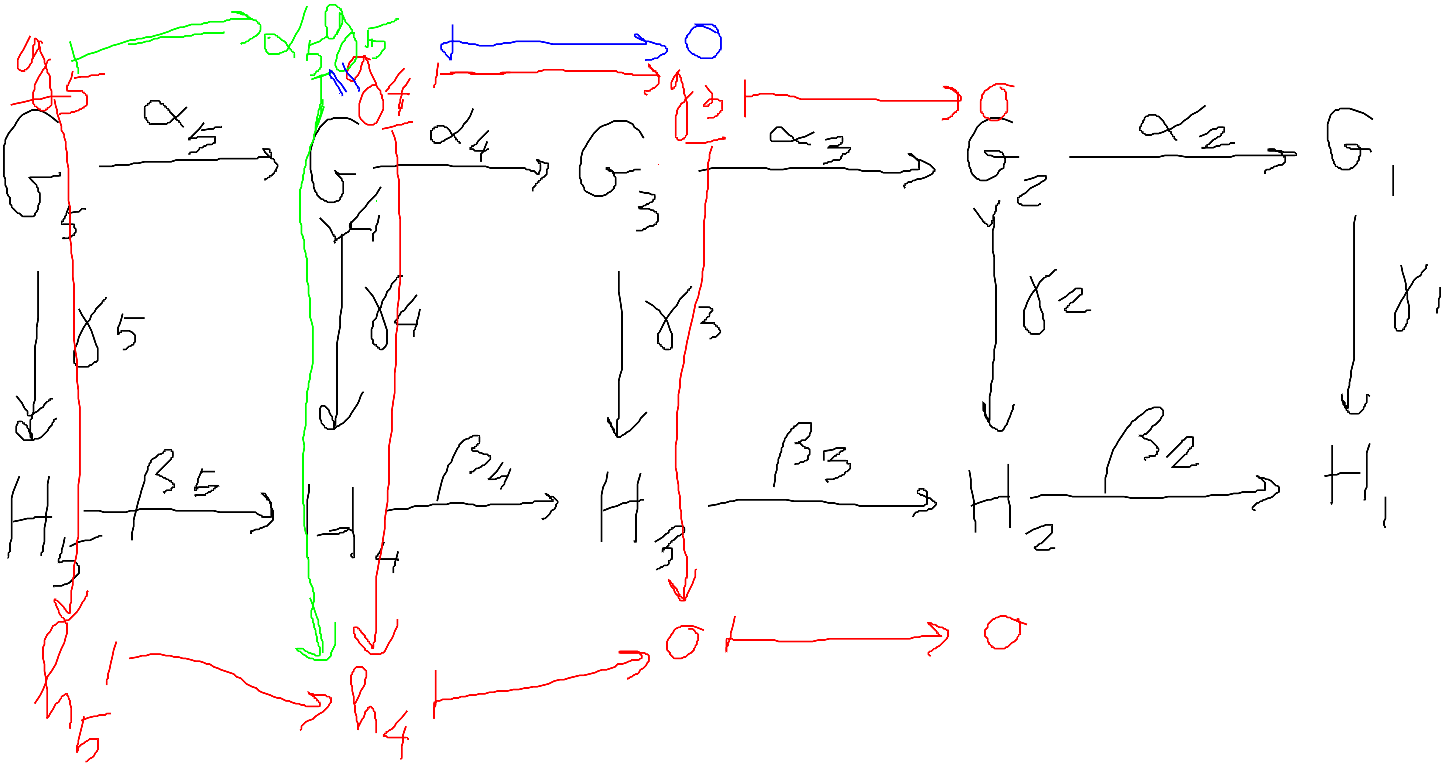
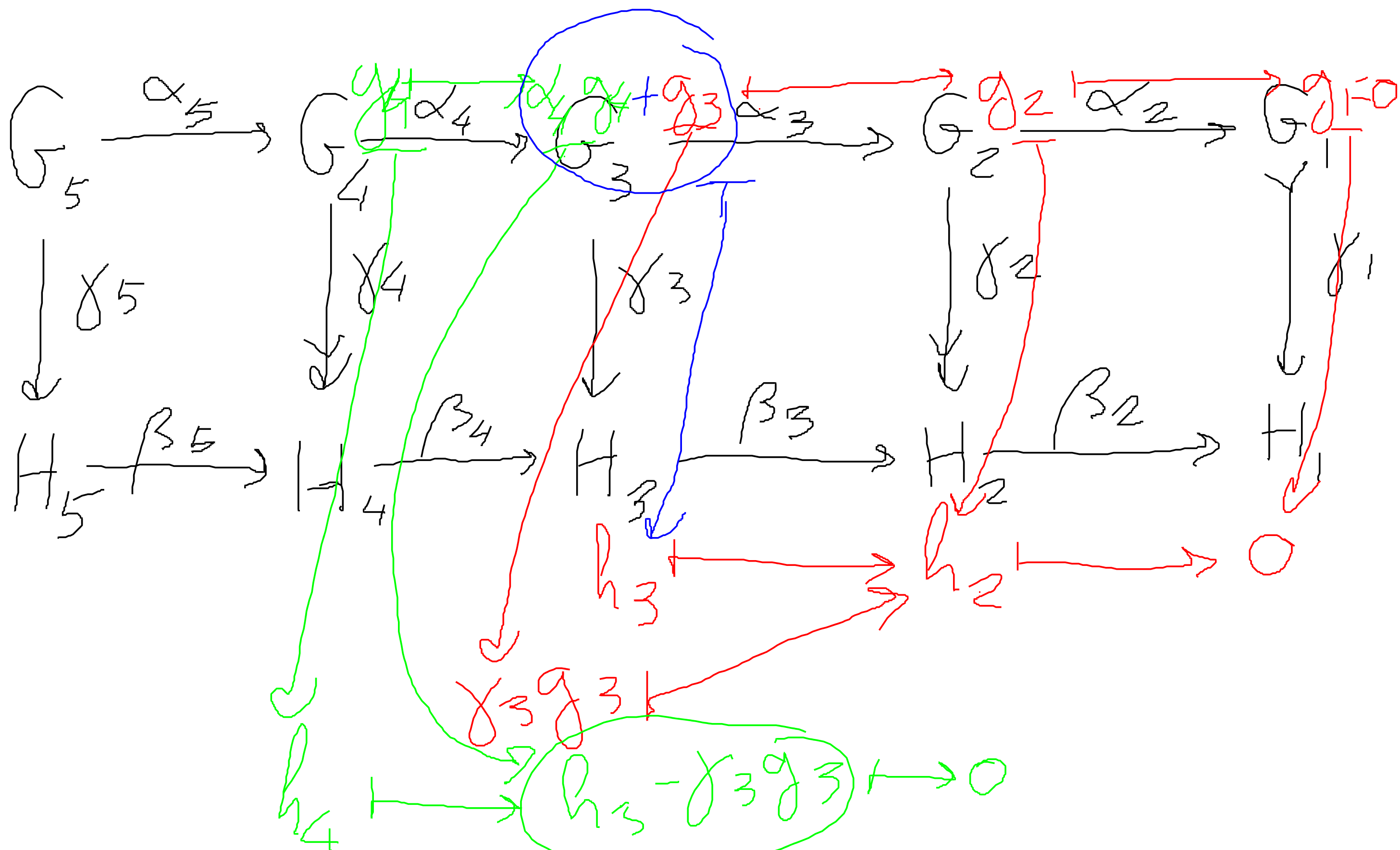




$$\gamma_3(\alpha_4 g_4 + g_3) = h_3 - \cancel{\gamma_3 g_3} + \cancel{\gamma_3 g_3} = h_3$$

$$\mathcal{S}_n(\bar{J}, J) \longrightarrow \mathcal{S}_{n-1}(\bar{J}, J) \longrightarrow \mathcal{S}_{n-2}$$

$$0 \longrightarrow \mathbb{Z} \longrightarrow 0 \longrightarrow 0$$

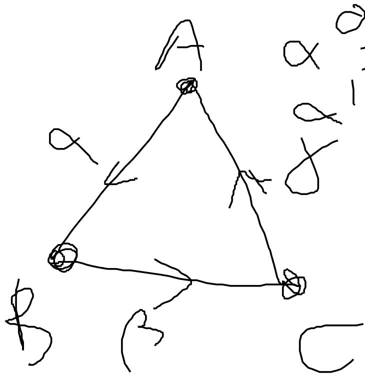
$$Z_n \cong \mathbb{Z}$$

$$B_n = 0$$

$$\begin{tikzcd}
\tilde{H}_k(x_0) \arrow[r] \arrow[d, phantom, "\|"] & \tilde{H}_k(X) \arrow[r] \arrow[rd, phantom, "\cong"] & H_k(X, x_0) \arrow[r] & \tilde{H}_{k-1}(x_0) \arrow[r] \arrow[d, phantom, "\|"] & {} \\
0 & & & 0 &
\end{tikzcd}$$

$$i : A \to B$$

$$\begin{tikzcd}
\tilde{H}_k(A) \arrow[r] \arrow[d, "i_*"] & \tilde{H}_k(X) \arrow[r] \arrow[d, "(1_X)_*"] & H_k(X, A) \arrow[r] \arrow[d, "(1_X)_*"] & \tilde{H}_{k-1}(A) \arrow[r] \arrow[d, "i_*"] & \tilde{H}_{k-1}(X) \arrow[d, "(1_X)_*"] \\
\tilde{H}_k(B) \arrow[r] & \tilde{H}_k(X) \arrow[r] & H_k(X, B) \arrow[r] & \tilde{H}_{k-1}(B) \arrow[r] & \tilde{H}_{k-1}(X)
\end{tikzcd}$$



$\alpha^0 = 3$

$\alpha^1 = 3$

$$E^0 = \begin{array}{c|ccc} & \alpha & \beta & \gamma \\ \hline A & -1 & 0 & 1 \\ B & 1 & -1 & 0 \\ C & 0 & 1 & -1 \end{array}$$

$\mathrm{II} \leftarrow \mathrm{II} + \mathrm{I}$

$$\rightsquigarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$\mathrm{III} \leftarrow \mathrm{III} + \mathrm{II}$

$$\rightsquigarrow \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \rightsquigarrow$$

$\mathrm{III} \leftarrow \mathrm{III} + \mathrm{I} + \mathrm{II}$ $\quad$ $\mathrm{I} \rightarrow \mathrm{II}$ $\quad$ $\mathrm{II} \leftrightarrow \mathrm{III}$

$\mathrm{I} \leftarrow -\mathrm{I}$

$\mathrm{II} \leftarrow -\mathrm{II}$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

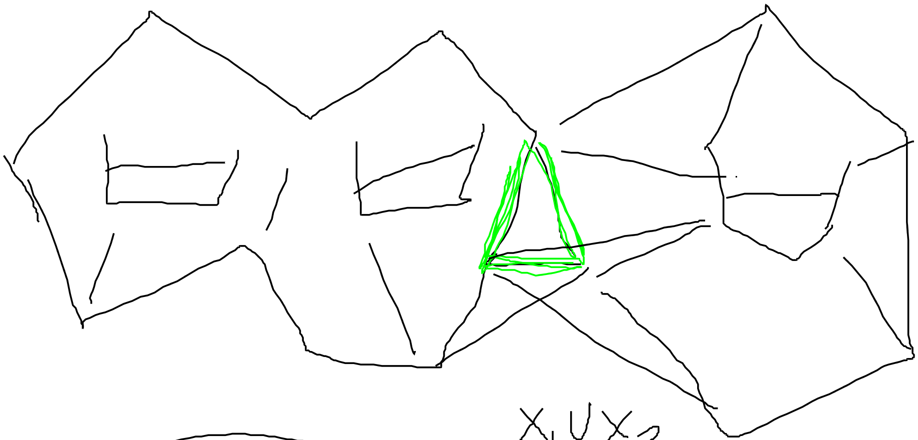
no torsione

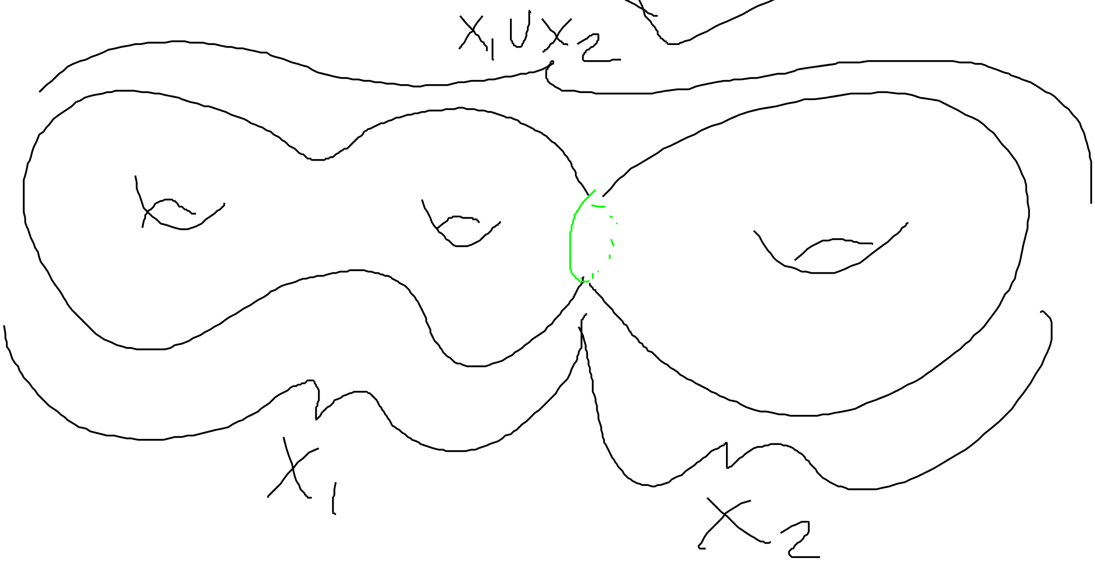
$\gamma^0 = 2$

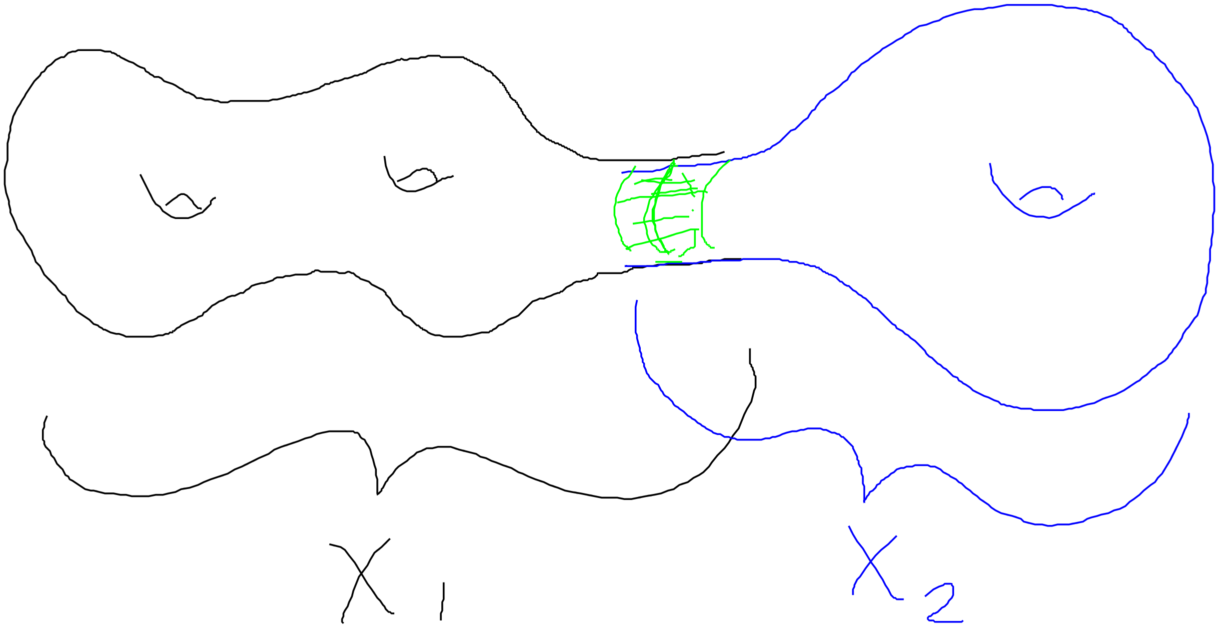
$$\beta_0 = \alpha^0 - \gamma^0 - \gamma^{-1} = 3 - 2 - 0 = 1$$

$$\beta_1 = \alpha^1 - \gamma^1 - \gamma^0 = 3 - 0 - 2 = 1$$

$$H_0(S^1) \simeq \mathbb{Z} \qquad H_1(S^1) \simeq \mathbb{Z}$$




$X_1 \cup X_2$
X_1
X_2



è un retratto forte per deformazione di X