TA200422

mercoledì 22 aprile 2020 08:41

$$\underbrace{\mathbb{C}^2 \setminus \{0\}}_{\cong} \supseteq \underbrace{S^3}_{E} \xrightarrow{p} \underbrace{\mathbb{C}P^1}_{,} \cong S^2$$

$$p^{-1}([w_0, u_0]) \overset{\circledast}{=} \{\lambda(w_0, u_0) : \lambda \in \mathbb{C}\ \|\lambda(w_0, u_0)\| = 1\} \quad \|(w_0, u_0)\| = 1$$

$$\|\lambda\| \cdot \underbrace{\|(w_0, u_0)\|}_{=1} = 1 \quad \|\lambda\| = 1$$

$$\lambda \in S^1$$

$$\circledast = \{\lambda(w_0, u_0) : \lambda \in S^1 \subseteq \mathbb{C}\} \cong S^1$$

NON E' UN RIVESTIMENTO PERCHE' LA FIBRA NON E' DISCRETA.

F insieme con top discreta

$x \in X \qquad p : \tilde{X} \longrightarrow X$

$x \in X$ $\qquad p : X \longrightarrow X$

$x \in U \qquad p^{-1}(U) \cong \underbrace{U \times F}$

PROIEZ RIVEST.
FIBRA
COSTANTE

$$\begin{tikzcd} p^{-1}(U) \arrow[r, "\cong"] \arrow[d, "p"'] & U \times F \arrow[dl, "p_1"] \\ U & \end{tikzcd}$$

MAPPA di HOPF FIBRATO
CON FIBRA S^1: $\quad p : E \xrightarrow{\text{continua}} B$
E' UN FIBRATO CON FIBRA F
SE:

$\forall x \in B \quad \exists\, U \ni x$

$$\begin{tikzcd} p^{-1}(U) \arrow[r, "\cong"] \arrow[d, "p"'] & U \times F \arrow[dl, "p_1"] \\ U & \end{tikzcd}$$

se B ha delle "buone" proprietà

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se B è PARACOMPATTO

$\forall$ RICOPRIMENTO APERTO ESISTE UN RAFFINAMENTO (UN RICOPRIMENTO APERTO I CUI APERTI SONO CONTENUTI IN QUELLO DI PARTENZA) LOCALMENTE FINITO.

UN FIBRATO AMMETTE SOLLEVAMENTI DI TUTTE LE OMOTOPIE

HP f' ESISTA

DIM TH 6.5

$$
\begin{tikzcd}
 & \pi_1(\tilde{X}, \tilde{x}_0) \arrow[d, "p_\#"] \\
\pi_1(Y, y_0) \arrow[ur, "f'_\#"] \arrow[r, "(f)_\#"'] \arrow[ur, phantom, "\equiv", shift left=6] & \pi_1(X, x_0)
\end{tikzcd}
$$

$$f_\# = p_\# \, f'_\#$$

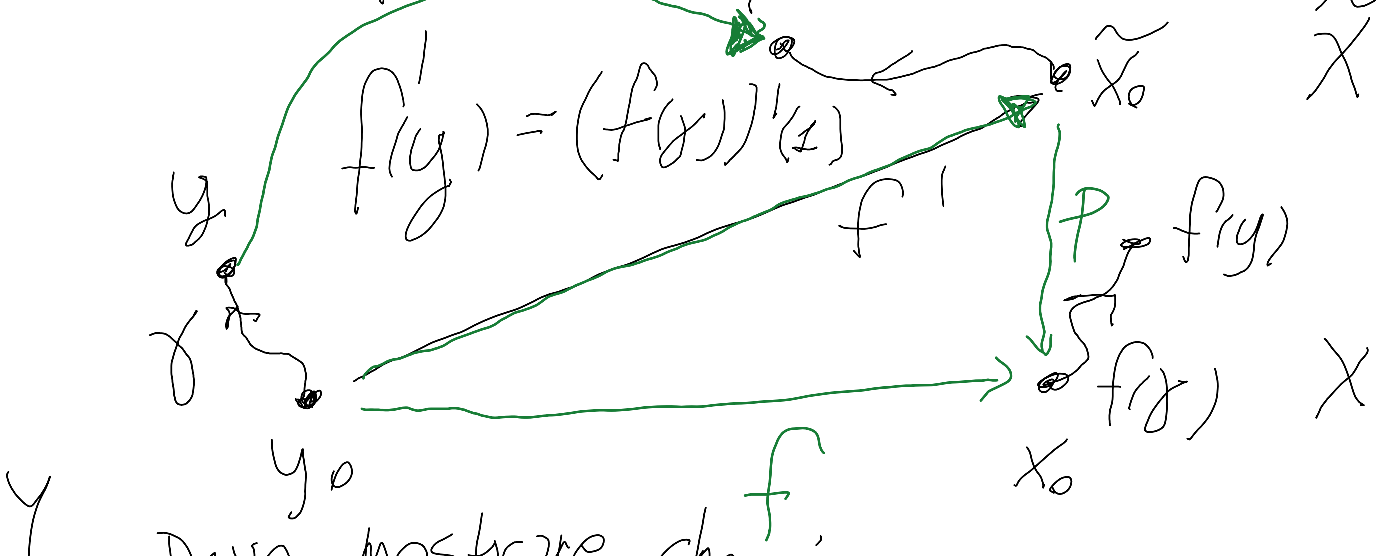
$$f_\# = p_\# f_\#$$

$$f_\#(\pi_1(Y, y_0)) = p_\#(f'_\#(\pi_1(Y, y_0)))$$

$$f_\#(\pi_1(Y, y_0)) \subseteq \mathrm{Im}\, p_\#$$

<u>HP</u> $f_\#(\pi_1(Y, y_0)) \subseteq p_\#(\pi_1(\tilde{X}, \tilde{x}_0))$

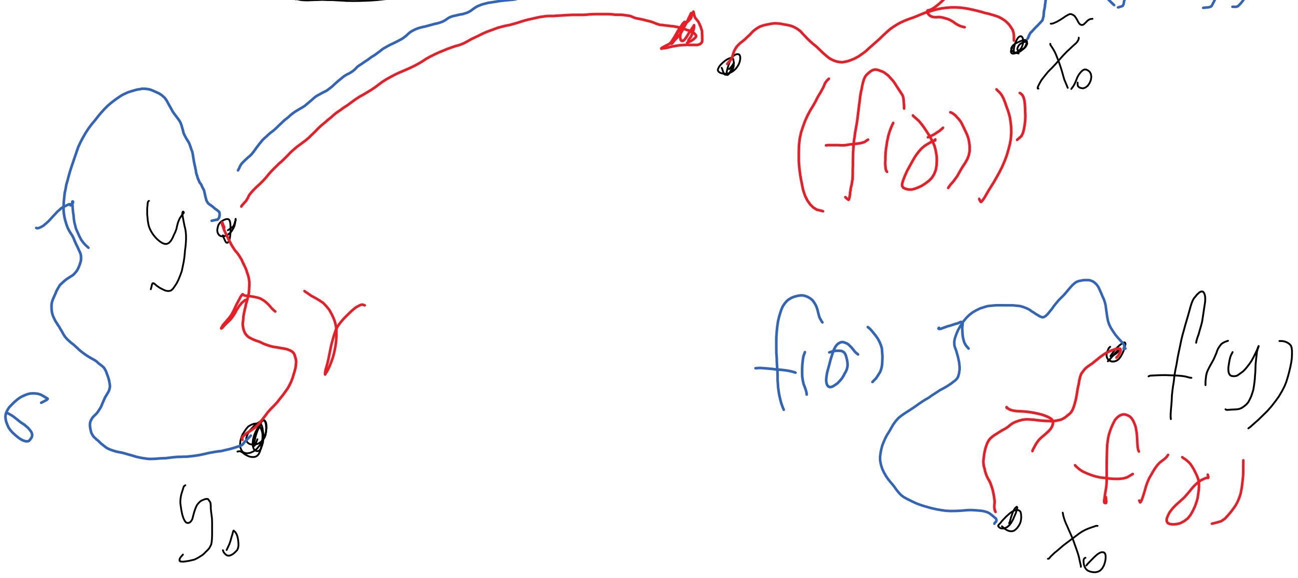
$$f'(y) = (\tilde{f}(\gamma))'(1)$$



Devo mostrare che f':

1. BEN DEFINITA: NON DIPENDE DALLA SCELTA di γ
2. CONTINUA
3. $f = p f'$ ✓

1. DISEGNO



$$[\gamma\sigma^{-1}] \in \pi_1(Y, y_0)$$

$$[f(\gamma\sigma^{-1})] \overset{HP}{\in} p_\#(\pi_1(\tilde{X}, \tilde{x}_0))$$

$\Longrightarrow (f(\gamma\sigma^{-1}))'$ CAPPIO BASATO SU $\tilde{x}_0$

$$(f(\gamma\sigma^{-1}))' = (f(\gamma)\cdot f(\sigma^{-1}))' =$$

$$= (f(\gamma))' \cdot (f(\sigma^{-1}))' =$$

$$= (f(\gamma))' \left((f(\sigma))' \right)^{-1}$$

$$\Longrightarrow (f(\gamma))(1) = (f(\sigma))(1)$$

2. CONTINUITÀ segue da loc c.p.a.
DALLE C.P.A.

$$p : (\tilde{X}, \tilde{x}_0) \longrightarrow (X, x_0)$$

X connesso loc. c.p.a

$\Downarrow$

$\tilde{X}$ connesso (loc. c.p.a.)

$$\left\{ p : (\tilde{X}, \tilde{x}_0) \longrightarrow (X, x_0) \right\} \Big/ \text{equival.}$$

$\updownarrow \; p_\#$

INIETTIVA? sì

SURIETTIVA? sì

$$\left\{ p_{\#}\left(\pi_1(\tilde{X}, \tilde{x}_0)\right) \leq \pi_1(X, x_0) \right\}$$

sottogruppo

$$\left.\begin{array}{l} \pi_1(X, x_0) \supseteq H \text{ sottogruppo} \\ \rightsquigarrow (\tilde{X}, \tilde{x}_0) \xrightarrow[p]{} (X, x_0) \\ \qquad\qquad\qquad\qquad\qquad \text{RIV.} \\ p_{\#}\left(\pi_1(\tilde{X}, \tilde{x}_0)\right) = H \end{array}\right\}$$

$H = \{e\}$ SOTTOGRUPPO BANALE

$$\rightsquigarrow \pi_1(\tilde{X}_{\{e\}}, \tilde{x}_0) = \{e\}$$

$\tilde{X}_{\{e\}}$ SEMPLICEMENTE CONNESSO

$\tilde{X}_{\{e\}}$

$$\begin{array}{c} \tilde{X} \text{ reg} \\ \big\downarrow \\ X \end{array}$$

U REG. RICOPERTO

$$\tilde{X} \supseteq \tilde{U} \supseteq \gamma(I)$$

NULLOM.

$$\tilde{U} \xrightarrow[\cong]{p} U$$

$$\underbrace{U \xhookrightarrow{i} X}$$

COMPONENDO L'OMOTOPIA CON p AVRÒ CHE ANCHE γ È NULLOMOTOPO IN X

$\Longrightarrow$ X SEMILOC 1-CONNESSO

$$(\tilde{X}_1, \tilde{x}_1) \xrightarrow[\sim]{f} (\tilde{X}_2, \tilde{x}_2)$$

$$
\begin{tikzcd}
(X_1, x_1) \arrow[rr, "\cong"] \arrow[dr, "p_1"'] & & (X_2, x_2) \arrow[dl, "p_2"] \\
& (X, x_0) &
\end{tikzcd}
$$

$$
\begin{aligned}
p: \; P(X, x_0)/\sim_H &\longrightarrow (X, x_0) \\
[\gamma] &\longmapsto \gamma(1)
\end{aligned}
$$

$$
p_{\#}\left(\pi_1\left(P(X, x_0)/\sim_H,\; c_{x_0}\right)\right) = H
$$

$$
\begin{aligned}
p: \; \mathbb{R} &\longrightarrow S^1 \\
x &\longmapsto e^{ix}
\end{aligned}
$$

$$x \longmapsto e^{\prime n}$$

$$p_{\#}(\pi_1(\mathbb{R}, 0)) = \{e\}$$

$$\cong P(S^1, 1)/_{\sim_{\{e\}}}$$