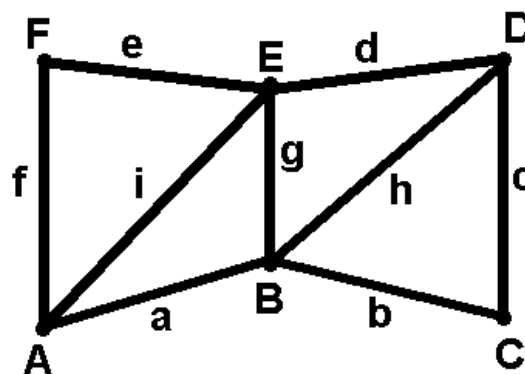


EXAMPLE 1

Let G be the graph drawn here:



1) (1 pt.) Adjacency matrix:

	A	B	C	D	E	F
A	0	1	0	0	1	1
B	1	0	1	1	1	0
C	0	1	0	1	0	0
D	0	1	1	0	1	0
E	1	1	0	1	0	1
F	1	0	0	0	1	0

2) (1 pt.) Incidence matrix:

	a	b	c	d	e	f	g	h	i
A	1	0	0	0	0	1	0	0	1
B	1	1	0	0	0	0	1	1	0
C	0	1	1	0	0	0	0	0	0
D	0	0	1	1	0	0	0	1	0
E	0	0	0	1	1	0	1	0	1
F	0	0	0	0	1	1	0	0	0

3) (1 pt.) Minimum degree $\delta = 2$ Maximum degree $\Delta = 4$

4) (1 pt.) Connectivity $\kappa = 2$ Edge-connectivity $\kappa' = 2$

5) (1 pt.) Is G bipartite? Why? (If answer is “yes”, list the two vertex sets of the bipartition)

No: It has odd cycles.

6) (1 pt.) Does G have an Euler tour? Why? (If answer is “yes”, write the edge sequence of one)

No: It has vertices with odd degree.

7) (1 pt.) Does G have an Euler trail? Why? (If answer is “yes”, write the edge sequence of one)

Yes: it has just two vertices with odd degree. abcdefigh

8) (1 pt.) Does G have a Hamilton path? (If answer is “yes”, write the vertex sequence of one)

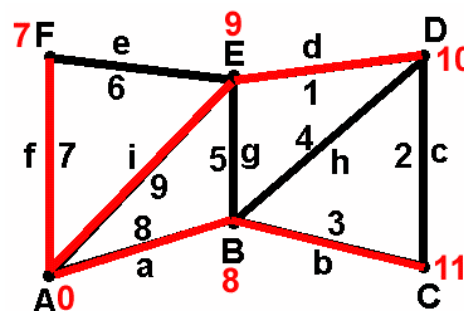
ABCDEF

9) (1 pt.) List the edge set of a maximum matching. Is it a perfect matching?

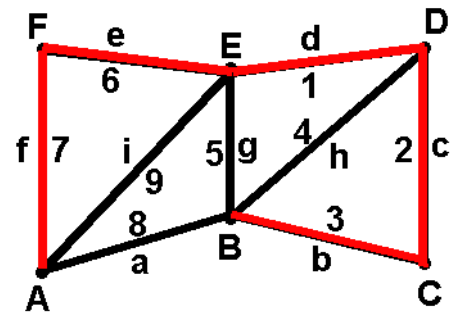
cgf Yes.

Now the vertices represent towns and the edge weights represent distances.

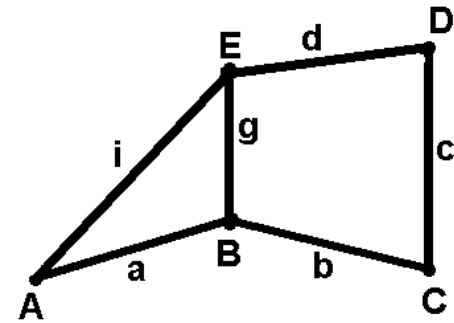
10) (2 pts.) Use Dijkstra’s algorithm to find minimal routes from A to all other vertices.



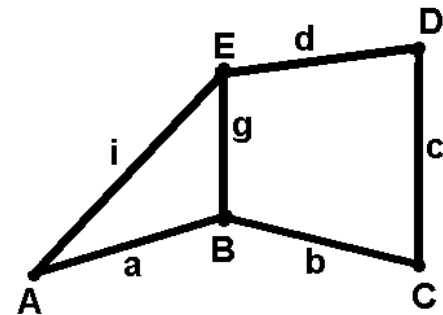
11) (2 pts.) Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) (3 pts.) Use the recursive formula to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 11



13) (4 pts.) Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).



$$\begin{aligned}
 & (A+BE)(B+AEC)(C+BD)(D+CE)(E+ABD) \\
 & (A+BE)(B+AEC) = AB+AAEC+BEB+BEAEC \\
 & (AB+AEC+BE)(C+BD) = \\
 & = ABC+AECC+BEC+ABBD+AECBD+BEBD \\
 & (ABC+AEC+BEC+ABD+BED)(D+CE) = \\
 & = ABCD+AECD+BECD+ABDD+BEDD+ABCCE+AECCE+BECCE+ABCDCE+BEDCE \\
 & (ABD+BED+AEC+BEC)(E+ABD) = \\
 & = ABDE+BEDE+AECE+BECE+ABDABD+BEDABD+AECABD+BECABD = \\
 & = BED+AEC+BEC+ABD
 \end{aligned}$$

Minimal coverings: {B,E,D}, {A,E,C}, {B,E,C}, {A,B,D}

Maximal independent sets: {A,C}, {B,D}, {A,D}, {C,E}

14) (4 pts.) Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).

$$k^5 - 6k^4 + 14k^3 - 15k^2 + 6k$$

