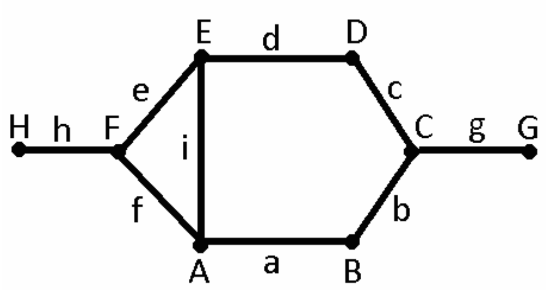
Mathematical Methods – 21 Dec. 2021 – Graph Theory
UniBo matriculation number:
(no name, please)

Let G be the graph drawn here:



1) *(1 pt.)* Adjacency matrix:

	A	B	C	D	E	F	G	H
A	0	1	0	0	1	1	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	1	0	0	1	0
D	0	0	1	0	1	0	0	0
E	1	0	0	1	0	1	0	0
F	1	0	0	0	1	0	0	1
G	0	0	1	0	0	0	0	0
H	0	0	0	0	0	1	0	0

2) *(1 pt.)* Incidence matrix:

	a	b	c	d	e	f	g	h	i
A	1	0	0	0	0	1	0	0	1
B	1	1	0	0	0	0	0	0	0
C	0	1	1	0	0	0	1	0	0
D	0	0	1	1	0	0	0	0	0
E	0	0	0	1	1	0	0	0	1
F	0	0	0	0	1	1	0	1	0
G	0	0	0	0	0	0	1	0	0
H	0	0	0	0	0	0	0	1	0

3) *(1 pt.)* Minimum degree δ = 1 Maximum degree Δ = 3

4) *(1 pt.)* Connectivity κ = 1 Edge-connectivity κ' = 1

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)
No. It contains odd cycles.

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)
No. It contains vertices of odd degree.

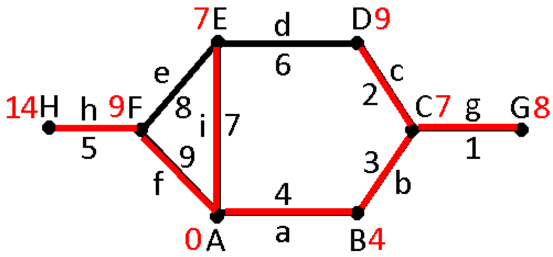
7) *(1 pt.)* Does G have an Euler trail? Why? (If answer is "yes", write the edge sequence of one)
No. It contains more than two vertices of odd degree.

8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)
No.

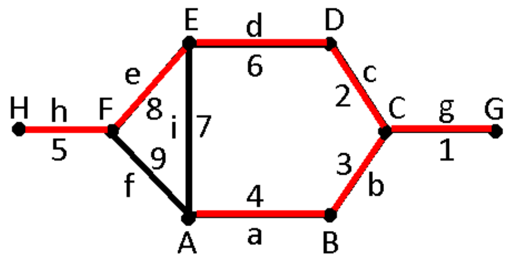
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?
{a,d,g,h} Yes.

Now the vertices represent towns and the edge weights represent distances.

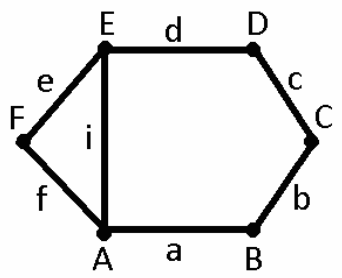
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



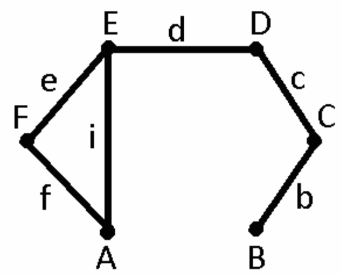
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).



12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 14



13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph (please show all passages).

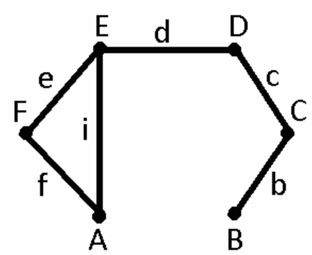


(A+EF)(B+C)(C+BD)(D+CE)(E+ADF)(F+AE) =
= (AB+AC+EFB+EFC)(C+BD)( ) =
= (ABC+ABBD+ACC+ACBD+EFBC+EFBBD+EFCC+EFCBD)( ) =
= (ABD+AC+EFBD+EFC)(D+CE)( ) =
= (ABDD+ABDCE+ACD+ACCE+EFBDD+EFBDCE+EFCD+EFCCE)( ) =
= (ABD+ACD+ACE+EFBD+EFC)(E+ADF)( ) =
= (ABDE+ABDADF+ACDE+ACDADF+ACEE+ACEADF+EFBDE+EFBDADF+EFCE+EFCADF)( ) =
= (ABDE+ABDF+ACDF+ACE+EFBD+EFC)(F+AE) =
= ABDEF+ABDEAE+ABDFF+ABDFAE+ACDFF+ACDFAE+ACEF+ACEAE+EFBDF+EFBDAE+EFCF+EFCAE =
= ABDE+ABDF+ACDF+ACE+BDEF+CEF

Minimal coverings: {A,B,D,E},{A,B,D,F},{A,C,D,F},{A,C,E},{B,D,E,F},{C,E,F}

Maximal independent sets: {C,F}, {C,E}, {B,E}, {B,D,F}, {A,C}, {A,B,D}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).



$k^6-6k^5+14k^4-16k^3+9k^2-2k$