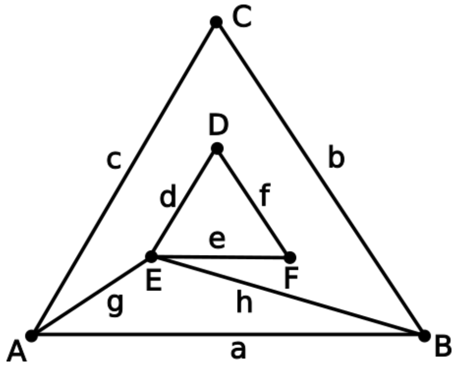
Mathematical Methods – 15 December 2023 – Graph Theory

**UniBo matriculation number:**

(no name, please)

Let G be the graph drawn here:




1) *(1 pt.)* Adjacency matrix:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| **A** | 0 | 1 | 1 | 0 | 1 | 0 |
| **B** | 1 | 0 | 1 | 0 | 1 | 0 |
| **C** | 1 | 1 | 0 | 0 | 0 | 0 |
| **D** | 0 | 0 | 0 | 0 | 1 | 1 |
| **E** | 1 | 1 | 0 | 1 | 0 | 1 |
| **F** | 0 | 0 | 0 | 1 | 1 | 0 |

2) *(1 pt.)* Incidence matrix:

| | a | b | c | d | e | f | g | h |
|---|---|---|---|---|---|---|---|---|
| **A** | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| **B** | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| **C** | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| **D** | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| **E** | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| **F** | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |

3) *(1 pt.)* Minimum degree $\delta =$ **2** Maximum degree $\Delta =$ **4**

4) *(1 pt.)* Connectivity $\kappa =$ **1** Edge-connectivity $\kappa' =$ **2**

5) *(1 pt.)* Is G bipartite? Why? (If answer is "yes", list the two vertex sets of the bipartition)

**No**: It has odd cycles.

6) *(1 pt.)* Does G have an Euler tour? Why? (If answer is "yes", write the edge sequence of one)

**No**: It has vertices with odd degree.

7) *(1 pt.)* Does G have an Euler trail with distinct origin and terminus? Why? (If answer is "yes", write the edge sequence of one)

**Yes**: it has exactly two vertices with odd degree. abcgefdh

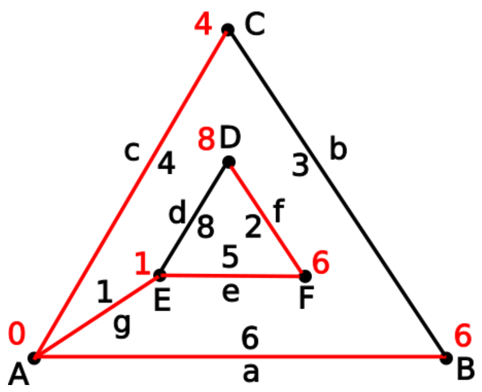
8) *(1 pt.)* Does G have a Hamilton path? (If answer is "yes", write the vertex sequence of one)

**Yes**. BCAEFD

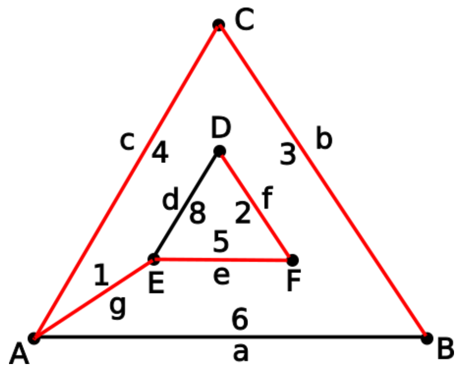
9) *(1 pt.)* List the edge set of a maximum matching. Is it a perfect matching?

bfg. Yes.

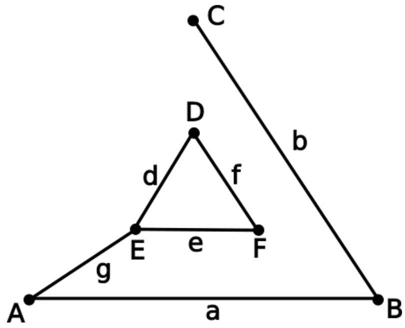
Now the vertices represent towns and the edge weights represent distances.
10) *(2 pts.)* Use Dijkstra's algorithm to find minimal routes from A to all other vertices.



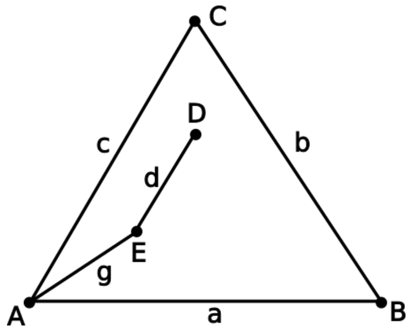
11) *(2 pts.)* Use Kruskal's algorithm to find a spanning tree with minimum total weight (an optimal connector of the towns).




12) *(3 pts.)* **Use the recursive formula** to compute τ (# of spanning trees) of this graph (passages not shown here, but in test you are supposed to show them): 3




13) *(4 pts.)* Use logic operations to find all minimal coverings and all maximal independent sets of this graph.




(A+BCE)(B+AC)(C+AB)(D+E)(E+AD) =
= (AB+AAC+BCEB+BCEAC)( ) = (AB+AC+BCE)(C+AB)( ) =
= (ABC+ABAB+ACC+ACAB+BCEC+BCEAB)( ) =
= (AB+AC+BCE)(D+E)( ) =
= (ABD+ABE+ACD+ACE+BCED+BCEE)( ) =
= (ABD+ABE+ACD+ACE+BCE)(E+AD) =
= ABDE+ABDAD+ABEE+ABEAD+ACDE+ACDAD+ACEE+ACEAD+BCEE+BCEAD =
= ABD+ABE+ACD+ACE+BCE

**Minimal coverings** : {A,B,D}, {A,B,E}, {A,C,D}, {A,C,E}, {B,C,E}
**Maximal independent sets** : {C,E}, {C,D}, {B,E }, {B,D}, {A,D}

14) *(4 pts.)* Compute the chromatic polynomial of this graph (passages not shown here, but in test you are supposed to show them).

**$k^6-6k^5+14k^4-16k^3+9k^2-2k$**