Lecture 2
Sheaves and Functors

In this lecture we introduce the basic concept of sheaf and we also recall some definitions and results of category theory.

1 Sheaves and locally ringed spaces

The definition of sheaf is central in algebraic and differential geometry since it provides a unified treatment. We start with two key examples.

Example 1.1. *Differentiable manifolds.* Let M be a differentiable manifold, whose topological space is Hausdorff and second countable. For each open set $U \subset M$, let $C^\infty(U)$ be the $\mathbf{R}$-algebra of smooth functions on U. The assignment:

$$U \longmapsto C^\infty(U)$$

satisfies the following two properties:

1. If $U \subset V$ are two open sets in M, we can define the *restriction* map:

$$\begin{array}{rcl} \mathrm{r}_{V,U} : C^\infty(V) & \longrightarrow & C^\infty(U) \\ f & \mapsto & f|_U \end{array}$$

which is such that:
i) $\mathrm{r}_{U,U} = \mathrm{id}$,
ii) $\mathrm{r}_{W,U} = \mathrm{r}_{V,U} \circ \mathrm{r}_{W,V}$, for $U \subset V \subset W$.

2. Let $\{U_i\}_{i \in I}$ be an open covering of U and let $\{f_i\}_{i \in I}$, $f_i \in C^\infty(U_i)$, be a family such that $f_i|_{U_i \cap U_j} = f_j|_{U_i \cap U_j}$ for all $i, j \in I$. In other words the elements of the family $\{f_i\}_{i \in I}$ agree on the intersection of any two open sets $U_i \cap U_j$. Then there exists a unique $f \in C^\infty(U)$ such that $f|_{U_i} = f_i$.

Such an assignment is called a *sheaf.* The pair (M, C^∞), consisting of the topological space M and the sheaf of the C^∞ functions on M is an example of *locally ringed space* (the word "locally" refers to a local property of the sheaf of C^∞ functions).

Given two manifolds M and N and the respective sheaves of smooth functions C^∞_M and C^∞_N, a morphism $f : M \longrightarrow N$ as ringed spaces, consists of a morphism $|f| : M \longrightarrow N$ of the underlying topological spaces together with a morphism of algebras:

$$f^* : C^\infty_N(V) \longrightarrow C^\infty_M(f^{-1}(V)), \qquad f^*(\phi)(x) = \phi(|f|(x)), \quad V \quad \text{open in} \quad |N|$$

compatible with the restriction morphisms.

Notice that, as soon as we give the continuos map $|f|$ between the topological spaces, the morphism f^* is automatically assigned. This is a peculiarity of the sheaf of smooth functions on a manifold. Such property is no longer true for a generic ringed space and, in particular, as we shall see, it is not true for supermanifolds.

A morphism of differentiable manifolds gives rise to a unique (locally) ringed space morphism and vice-versa.

Moreover, given two manifolds, they are isomorphic as manifolds if and only if they are isomorphic as (locally) ringed spaces. In the language of categories, we say we have a fully faithfull functor from the category of manifolds to the category of locally ringed spaces.

Before going to the general treatment, let us consider another interesting example arising from classical algebraic geometry.

Example 1.2. *Algebraic varieties.* Let X be an affine algebraic variety in the affine space $\mathbf{A}^n$ over an algebraically closed field k and let $\mathcal{O}(X) = k[x_1 \dots x_n]/I$ be its coordinate ring, where the ideal I is prime. This corresponds topologically to the irreducibility of the variety X. We can think to the points of X as the zeros of the polynomials in the ideal I in $\mathbf{A}^n$. X is a topological space with respect to the Zariski topology, whose closed sets are the zeros of the polynomials in the ideals of $\mathcal{O}(X)$. For each open U in X, consider the assigment:

$$U \longmapsto \mathcal{O}_X(U)$$

where $\mathcal{O}_X(U)$ is the k-algebra of of algebraic functions on U. By definition, these are the functions $f : X \longrightarrow k$ that can be expressed as a quotient of two polynomials at each point of $U \subset X$. As in the case of differentiable manifolds, our assignment satisfies the properties (1) and (2) described above. The first property is clear, while for the second one, we leave it as a (hard) exercise.

Hence $\mathcal{O}_X$ is a sheaf called the *structure sheaf* of the variety X of the sheaf of regular functions and $(X, \mathcal{O}_X)$ is another example of (locally) ringed space.

We are now going to formulate more generally the notion of sheaf and of ringed space.

Definition 1.3. Let $|M|$ be a topological space. A *presheaf* of commutative algebras $\mathcal{F}$ on X is an assignment

$$U \longmapsto \mathcal{F}(U), \quad U \quad \text{open in } |M|, \quad \mathcal{F}(U) \quad \text{a commutative algebra},$$

such that:
1. If $U \subset V$ are two open sets in $|M|$, there exists a morphism
$\mathrm{r}_{V,U} : \mathcal{F}(V) \longrightarrow \mathcal{F}(U)$, called the *restriction morphism* and often denoted with $\mathrm{r}_{V,U}(f) = f|_U$, such that:
i) $\mathrm{r}_{U,U} = \mathrm{id}$,
ii) $\mathrm{r}_{W,U} = \mathrm{r}_{V,U} \circ \mathrm{r}_{W,V}$, for $U \subset V \subset W$.

The presheaf $\mathcal{F}$ is called a *sheaf* if:
2. Given $\{U_i\}_{i\in I}$, an open covering of U and a family $\{f_i\}_{i\in I}$, $f_i \in \mathcal{F}(U_i)$, such that $f_i|_{U_i\cap U_j} = f_j|_{U_i\cap U_j}$ for all $i, j \in I$, there exists a unique $f \in \mathcal{F}(U)$ such that $f|_{U_i} = f_i$.

The elements in $\mathcal{F}(U)$ are called *sections* over U; when $U = |M|$ we call such elements *global sections.*

A most important object associated to a given presheaf is the *stalk* at a point.

Definition 1.4. Let $\mathcal{F}$ be a presheaf on the topological space $|M|$ and let x be a point in $|M|$. We define the *stalk* $\mathcal{F}_x$ of $\mathcal{F}$, at the point x, as the inductive limit:

$$\lim_{\rightarrow} \mathcal{F}(U),$$

where the direct limit is taken for all the U open neighbourhoods of x in $|M|$. $\mathcal{F}_x$ consists of the disjoint union of all pairs (U, s) with U open in $|M|$, $x \in U$, and $s \in \mathcal{F}(U)$, modulo the equivalence relation: $(U, s) \cong (V, t)$ if and only if there exists a neighbourhood W of x, $W \subset U \cap V$, such that $s|_W = t|_W$.

The elements in $\mathcal{F}_x$ are called *germs of sections.*

Definition 1.5. Let $\mathcal{F}$ and $\mathcal{G}$ be presheaves on $|M|$. A *morphism of presheaves* $\phi : \mathcal{F} \longrightarrow \mathcal{G}$ is a collection of morphisms $\phi_U : \mathcal{F}(U) \longrightarrow \mathcal{G}(U)$, for each open set U in $|M|$, such that for all $V \subset U$ the following diagram commutes:

$$\begin{tikzcd}
\mathcal{F}(U) \arrow[r, "\phi_U"] \arrow[d, "\mathrm{r}_{UV}"'] & \mathcal{G}(U) \arrow[d, "\mathrm{r}_{UV}"] \\
\mathcal{F}(V) \arrow[r, "\phi_V"] & \mathcal{G}(V)
\end{tikzcd}$$

A *morphisms of sheaves* is just a morphism of the underlying presheaves.

Clearly any morphism of presheaves induces a morphism on the stalks: $\phi_x : \mathcal{F}_x \longrightarrow \mathcal{G}_x$. The sheaf property, i.e. property (2) in the definition 1.3, ensures that, if we have two morphisms of sheaves ϕ and ψ such that $\phi_x = \psi_x$ for all x, then $\phi = \psi$.

We say that the morphism of sheaves ϕ is *injective (resp. surjective)* if ϕ_x is injective (resp. surjective).

On the notion of surjectivity, however, one should exert some care, since we can have a surjective sheaf morphism $\phi : \mathcal{F} \longrightarrow \mathcal{G}$ such that $\phi_U : \mathcal{F}(U) \longrightarrow \mathcal{G}(U)$ is *not* surjective for some open sets U. This strange phenomenon is a consequence of the following fact. While the assignment $U \mapsto ker(\phi(U))$ always defines a sheaf, the assignment $U \mapsto Im(\phi(U))$ defines in general only a presheaf. In order to define the image sheaf we need to do the *sheafification* of this presheaf. Intuitively, one may think to the sheafification as the sheaf that best "approximates" the given presheaf. We shall not pursue furtherly this point.

We are ready for the definition of locally ringed space. This definition is very important for us, since its supergeometric correspondent, that we shall introduce in the next lecture, is essential to define supermanifolds and superschemes, which are the basic ingredients of supergeometry.

Definition 1.6. We define *ringed space* a pair $M = (|M|, \mathcal{F})$ consisting of a topological space $|M|$ and a sheaf of commutative rings $\mathcal{F}$ on $|M|$. We say that the ringed space $(|M|, \mathcal{F})$ is a *locally ringed space*, if the stalk $\mathcal{F}_x$ is a local ring for all $x \in |M|$. A *morphism* of ringed spaces $\phi : M = (|M|, \mathcal{F}) \longrightarrow N = (|N|, \mathcal{G})$ consists of a morphism $|\phi| : |M| \longrightarrow |N|$ of the topological spaces (in other words, $|\phi|$ is a continuous map) and a sheaf morphism $\phi^* : \mathcal{O}_N \longrightarrow \phi_* \mathcal{O}_M$ where $\phi_* \mathcal{O}_M$ is the sheaf on $|N|$ defined as follows: $(\phi_* \mathcal{O}_M)(U) = \mathcal{O}_M(\phi^{-1}(U))$ for all U open in $|N|$. A morphism of ringed spaces induces a morphism on the stalks for each $x \in |M|$: $\phi_x : \mathcal{O}_{N,|\phi|(x)} \longrightarrow \mathcal{O}_{M,x}$. If M and N are locally ringed spaces, we say that the morphism of ringed spaces ϕ is a *morphism of locally ringed spaces* if ϕ_x is local, i.e. $\phi_x^{-1}(m_{M,x}) = m_{N,|\phi|(x)}$ where $m_{N,|\phi|(x)}$ and $m_{M,x}$ are the maximal ideals in the local rings $\mathcal{O}_{N,|\phi|(x)}$ and $\mathcal{O}_{M,x}$ respectively.

Observation 1.7. In the previous section we have seen differentiable manifolds and algebraic varieties as examples of ringed spaces. Actually both are also examples of locally ringed spaces, as one can readily verify. Moreover, one can also check that their morphisms, in the differential or in the algebraic setting respectively, correspond precisely to morphisms as locally ringed spaces.

At this point it is not hard to convince ourselves that we can take a different point of view on the definition of differentiable manifold. Namely we can equivalently define a differentiable manifold as a ringed space $M = (|M|, \mathcal{O}_M)$ as follows.

Definition 1.8. *Alternative definition of differentiable manifold.* Let M be a topological space, Hausdorff and second countable, and let $\mathcal{O}_M$ be a sheaf of commutative algebras on M, so that $(M, \mathcal{O}_M)$ is a locally ringed space. We say that $(M, \mathcal{O}_M)$ is a *real differentiable manifold* if it is locally isomorphic to the locally ringed space $(\mathbb{R}^n, C^\infty_{\mathbb{R}^n})$, where $C^\infty_{\mathbb{R}^n}$ is the sheaf of smooth functions on $\mathbf{R}^n$.

In the same way we can define analytic real or complex manifolds as locally ringed spaces locally isomorphic to $(\mathbf{R}^n, \mathcal{H}_{\mathbf{R}^n})$ or $(\mathbf{C}^n, \mathcal{H}_{\mathbf{C}^n})$, where $\mathcal{H}_{\mathbf{R}^n}$ and $\mathcal{H}_{\mathbf{C}^n}$ denote the sheaves of analytic functions on $\mathbf{R}^n$ or $\mathbf{C}^n$ respectively (we leave to the reader as an exercise their definition, see [2] for more details).

2 The functor of points

When we are dealing with classical manifolds we can altogether avoid the use of their functor of points. In fact they are well understood just by looking at their underlying topological spaces and the regular functions on the open sets.

The functor of points, originally introduced as a tool in algebraic geometry, can actually be employed in a much wider context. [1]

---

[1] Grothendieck idea behind the definition of the functor of points in algebraic geometry is the following. If X is a scheme, for each commutative ring A, we can define the set of the A-points of X in analogy to the way the classical geometers used to define the rational or integral points on a variety. The crucial difference is that we do not focus on just one commutative ring A, but we consider the A-points for all commutative rings A. In fact, the scheme we start from, is completely recaptured only by the collection of the A-points for *every* commutative ring A, together with the admissible morphisms.

Definition 2.1. Let $M = (|M|, \mathcal{O}_M)$ be a manifold and let (mflds) denote the category of manifolds. We define the *functor of points of manifold M* as the representable functor:

$$h_M : (\text{mflds})^o \longrightarrow (\text{sets}), \quad h_M(T) = \text{Hom}(T, M).$$

h_M on morphisms is defined as $h_M(\phi)(f) = f \circ \phi$.

We have the following important characterization of morphisms.

Proposition 2.2. *Let M and N be differentiable manifolds. Then:*

$$Hom(M, N) \cong Hom(C^\infty(N), C^\infty(M)).$$

We are now going to state Yoneda's Lemma, a basic categorical result. As an immediate consequence, we have that the functor of points of a scheme (resp. differentiable manifold) does determine the scheme (resp. differentiable manifold) itself.

Theorem 2.3. *Yoneda's Lemma.*
Let $\mathcal{C}$ be a category and let X, Y be objects in $\mathcal{C}$ and let $h_X : \mathcal{C}^o \longrightarrow (\text{sets})$ be the representable functor defined on the objects as $h_X(T) = Hom(T, X)$ and, as usual, on the arrows as $h_X(\phi)(f) = f \cdot \phi$, for $\phi : T \longrightarrow S$, $f \in Hom(T, S)$.

1. If $F : \mathcal{C}^o \longrightarrow (\text{sets})$, then we have a one to one correspondence between the sets:

$$\{h_X \longrightarrow F\} \quad \Longleftrightarrow \quad F(X)$$

2. The functor

$$\begin{array}{ccc} h : \mathcal{C} & \longrightarrow & Fun(\mathcal{C}^o, (\text{sets})) \\ X & \mapsto & h_X \end{array}$$

is an equivalence of $\mathcal{C}$ with a full subcategory of functors. In particular, $h_X \cong h_Y$ if and only if $X \cong Y$ and the natural transformations $h_X \longrightarrow h_Y$ are in one to one correspondence with the morphisms $X \longrightarrow Y$.

Proof. We briefly sketch it, leaving the details to the reader. Let $\alpha : h_X \longrightarrow F$. We can associate to α, $\alpha_X(\text{id}_X) \in F(X)$. Vice-versa, if $p \in F(X)$, we associate to p, $\alpha : h_X \longrightarrow F$ such that

$$\alpha_Y : \text{Hom}(Y, X) \longrightarrow F(Y), \qquad f \longmapsto F(f)p.$$

■

Corollary 2.4. *Two manifolds are isomorphic if and only if their functor of points are isomorphic.*

Example 2.5. Let us take $M = \mathbf{R}^n$. Then the T-points of M are given by:

$$h_{\mathbf{R}^n}(T) = \mathrm{Hom}(T, \mathbf{R}^n) = \mathrm{Hom}(C^\infty(\mathbf{R}^n), C^\infty(T)) = C^\infty(T)^n$$

since any morphism $C^\infty(\mathbf{R}^n) \longrightarrow C^\infty(T)$ is determined once we know the images of the n global coordinates in $C^\infty(\mathbf{R}^n)$ in other words once we know an n-uple $(t_1 \dots t_n) \subset C^\infty(T)^n$. Hence the T-points of $\mathbf{R}^n$ are identified with n-uples of global functions in $C^\infty(T)$. Similarly $h_{\mathrm{M}_n}(T)$ the T points of the $n \times n$ matrices are identified with $n \times n$ matrices with coefficients in $C^\infty(T)$, while $h_{\mathrm{GL}_n}(T)$ with $n \times n$ *invertible* matrices with coefficients in $C^\infty(T)$.

The advantages of using the functorial language are many. Morphisms of schemes are just maps between the sets of their A-points, respecting functorial properties. This often simplifies matters, allowing to leave the sheaves machinery on the background.

3 Appendix: Categories and functors

We want to make a brief summary of formal properties and definitions relative to categories.

Definition 3.1. A *category* $\mathcal{C}$ consists of a collection of objects, denoted by $Ob(\mathcal{C})$, and sets of *morphisms* between objects. For all pairs $A, B \in Ob(\mathcal{C})$, we denote the set of morphisms from A to B by $\mathrm{Hom}_{\mathcal{C}}(A, B)$ so that for all $A, B, C \in \mathcal{C}$, there exists an association

$$\mathrm{Hom}_{\mathcal{C}}(B, C) \times \mathrm{Hom}_{\mathcal{C}}(A, B) \longrightarrow \mathrm{Hom}_{\mathcal{C}}(A, B)$$

called the "composition law" $((f, g) \to f \circ g)$ which satisfies the properties
(i) the law "$\circ$" is associative,
(ii) for all $A, B \in Ob(\mathcal{C})$, there exists $id_A \in \mathrm{Hom}_{\mathcal{C}}(A, A)$ so that we get $f \circ id_A = f$ for all $f \in \mathrm{Hom}_{\mathcal{C}}(A, B)$ and $id_A \circ g = g$ for all $g \in \mathrm{Hom}_{\mathcal{C}}(B, A)$,
(iii) $\mathrm{Hom}_{\mathcal{C}}(A, B)$ and $\mathrm{Hom}_{\mathcal{C}}(A', B')$ are disjoint unless $A = A'$, $B = B'$ in which case they are equal.

If a morphism $f \in \mathrm{Hom}_{\mathcal{C}}(A, B)$ is invertible, in other words there exist another morphism $g \in \mathrm{Hom}_{\mathcal{C}}(B, A)$ such that $f \circ g$ and $g \circ f$ are the identities respectively in $\mathrm{Hom}_{\mathcal{C}}(B,B)$ and $\mathrm{Hom}_{\mathcal{C}}(A,A)$, we say that f is an *isomorphism.*

Once the category is understood, it is conventional to write $A \in \mathcal{C}$ instead of $A \in Ob(\mathcal{C})$ for objects. We may also suppress the "$\mathcal{C}$" from $\mathrm{Hom}_{\mathcal{C}}$ and just write Hom whenever there is no danger of confusion.

Essentially a category is a collection of objects which share some basic structure, along with maps between objects which preserve that structure.

Example 3.2. 1. Let (sets) denote the category of sets. The objects are the sets, and for any two sets $A, B \in Ob((\text{sets}))$, the morphisms are the maps from A to B.

2. Let $\mathcal{G}$ denote the category of groups. Any object $G \in \mathcal{G}$ is a group, and for any two groups $G, H \in Ob(\mathcal{G})$, the set $\mathrm{Hom}_{\mathcal{G}}(G,H)$ is the set of group homomorphisms from G to H.

Definition 3.3. A category $\mathcal{C}'$ is a *subcategory* of category $\mathcal{C}$ if $Ob(\mathcal{C}') \subset Ob(\mathcal{C})$ and if for all $A, B \in \mathcal{C}'$, $\mathrm{Hom}_{\mathcal{C}'}(A,B) \subset \mathrm{Hom}_{\mathcal{C}}(A,B)$ so that the composition law "$\circ$" on $\mathcal{C}'$ is induced by that on $\mathcal{C}$.

Example 3.4. The category $\mathcal{A}$ of *abelian groups* is a subcategory of the category of groups $\mathcal{G}$.

Definition 3.5. Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two categories. Then a *covariant [contravariant] functor* $F : \mathcal{C}_1 \longrightarrow \mathcal{C}_2$ consists of
(1) a map $F : Ob(\mathcal{C}_1) \longrightarrow Ob(\mathcal{C}_2)$ and
(2) a map (denoted by the same F) $F : \mathrm{Hom}_{\mathcal{C}_1}(A,B) \longrightarrow \mathrm{Hom}_{\mathcal{C}_2}(F(A),F(B))$ $[F : \mathrm{Hom}_{\mathcal{C}_1}(A,B) \longrightarrow \mathrm{Hom}_{\mathcal{C}_2}(F(B),F(A))]$ so that
(i) $F(id_A) = id_{F(A)}$ and
(ii) $F(f \circ g) = F(f) \circ F(g)$ $[F(f \circ g) = F(g) \circ F(f)]$
for all $A, B \in Ob(\mathcal{C}_1)$.

When we say "functor" we mean covariant functor. A contravariant functor $F : \mathcal{C}_1 \longrightarrow \mathcal{C}_2$ is the same as a covariant functor from $\mathcal{C}_1^o \longrightarrow \mathcal{C}_2$ where $\mathcal{C}_1^o$ denotes the *opposite* category i. e. the category where all morphism arrows are reversed.

Definition 3.6. Let F_1, F_2 be two functors from $\mathcal{C}_1$ to $\mathcal{C}_2$. We say that there is a *natural transformation* of functors $\varphi : F_1 \longrightarrow F_2$ if for all $A \in \mathcal{C}_1$ there is a set of morphisms $\varphi_A : F_1(A) \longrightarrow F_2(A)$ so that for any $f \in \mathrm{Hom}_{\mathcal{C}_1}(A,B)$ ($B \in \mathcal{C}_1$), the following diagram commutes:

$$\begin{tikzcd} F_1(A) \arrow[r, "\varphi_A"] \arrow[d, "F_1(f)"'] & F_2(A) \arrow[d, "F_2(f)"] \\ F_1(B) \arrow[r, "\varphi_B"] & F_2(B). \end{tikzcd} \tag{1}$$

We say that the family of functions φ_A is *functorial* in A.

We say that two functors $F, G : \mathcal{C}_1 \longrightarrow \mathcal{C}_2$ are *isomorphic* if there exist two natural transformations $\phi : F \longrightarrow G$ and $\psi : G \longrightarrow F$ such that $\phi \circ \psi = \mathrm{id}$ and $\psi \circ \phi = \mathrm{id}$.

The functors from $\mathcal{C}_1$ to $\mathcal{C}_2$ for any two given categories together with their natural transformations form a category.

The notion of equivalence of categories is important since it allows to identify two categories which are apparently different.

Definition 3.7. We say that two categories $\mathcal{C}_1$ and $\mathcal{C}_2$ are *equivalent* if there exists two functors $F : \mathcal{C}_1 \longrightarrow \mathcal{C}_2$ and $G : \mathcal{C}_1 \longrightarrow \mathcal{C}_2$ such that $FG \cong id_{\mathcal{C}_2}$, $GF \cong id_{\mathcal{C}_1}$ (where $id_{\mathcal{C}}$ denotes the identity functor of a given category, defined in the obvious way).

If F is a functor from the category $\mathcal{C}_1$ to the category $\mathcal{C}_2$, for any two objects A, $B \in \mathcal{C}_1$, by its very definition F induces a function (that we denoted previously with F):

$$F_{A,B} : \mathrm{Hom}_{\mathcal{C}_1}(A,B) \longrightarrow \mathrm{Hom}_{\mathcal{C}_2}(F(A),F(B)).$$

Definition 3.8. Let F be a functor. We say that F is *faithful* if $F_{A,B}$ is injective, we say F is *full* if $F_{A,B}$ is surjective and we say that F is *fully faithful* if $F_{A,B}$ is bijective.

Next we want to formally define what it means for a functor to be *representable*. Let us first define the representation functors.

Definition 3.9. Let $\mathcal{C}$ be a category, A a fixed object in $\mathcal{C}$. We define the two *representation functors* Hom^A, Hom_A as

$$\begin{aligned} \mathrm{Hom}^A : \mathcal{C}^o \longrightarrow (\text{sets}), \qquad & B \mapsto \mathrm{Hom}_{\mathcal{C}}(B,A) \\ \mathrm{Hom}_A : \mathcal{C} \longrightarrow (\text{sets}), \qquad & B \mapsto \mathrm{Hom}_{\mathcal{C}}(A,B) \end{aligned}$$

where (sets) denotes the category of sets. On the arrow $f \in \mathrm{Hom}(B, C)$ we have:

$$\mathrm{Hom}^A(f)\phi = \phi \circ f, \quad \phi \in \mathrm{Hom}^A(B) \qquad \mathrm{Hom}_A(f)\psi = f \circ \psi, \quad \psi \in \mathrm{Hom}_A(B).$$

Definition 3.10. Let F be a functor from the category $\mathcal{C}$ to the category of sets. We say that F is *representable by* $X \in \mathcal{C}$ if for all $A \in \mathcal{C}$, $F \cong \mathrm{Hom}_A$ or $F \cong \mathrm{Hom}^A$.

References

[1] D. Eisenbud and J. Harris, *The geometry of schemes.* Springer Verlag, New York, 2000.

[2] P. Griffiths, J. Harris, *Principles of Algebraic Geometry*, Wiley Classics Library, 1994.

[3] R. Hartshorne. *Algebraic geometry.* Graduate Text In Mathematics. Springer-Verlag, New York, 1977.