- My main research interests -
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I am mainly interested in comparing continuous real-valued functions with respect to the action of a group G of self-homeomorphisms of the topological space X where the functions are defined. If you are interested in applied mathematics, you can see these functions as physical or perceptual measurements that are compared to each other in order to check how similar they are with respect to the invariance expressed by the group G. As an example, you can think of two gray level images represented by two functions $\varphi, \psi : \mathbb{R}^2 \to \mathbb{R}$ and of their comparison with respect to the group of rigid motions of the real plane.

The main mathematical tool in this topic is the *natural pseudo-distance* d_G associated with a subgroup G of the group $\mathrm{Homeo}(X)$ of all self-homeomorphisms of a topological space X: for $\varphi, \psi : X \to \mathbb{R}$ we set

$$d_G(\varphi, \psi) := \inf_{g \in G} \|\varphi - \psi \circ g\|_\infty.$$

My main mathematical interest consists in studying the properties of d_G, from the topological and differential point of view.

The natural pseudo-distance d_G is quite difficult to study in a direct way. Fortunately, persistent homology can be used to get information about d_G. I started to do that in my PhD thesis (*Omotopie e invarianti metrici per sottovarietà di spazi euclidei (teoria della taglia)*, 1991) and in these three papers, using the concept of *size function* (i.e. persistent homology in degree 0):

- P. Frosini, *A distance for similarity classes of submanifolds of a Euclidean space*, Bulletin of the Australian Mathematical Society, 42, 3 (1990), 407-416.
- P. Frosini, *Measuring shapes by size functions*, Proc. of SPIE, Intelligent Robots and Computer Vision X: Algorithms and Techniques, Boston, MA, 1607 (1991), 122-133.
- P. Frosini, *Discrete computation of size functions*, Journal of Combinatorics, Information & System Sciences, 17, 3-4 (1992), 232-250.

However, persistent homology has to be adapted to the group G, in order to get better information about d_G. This observation leads to the concept of *G-invariant persistent homology*:

- P. Frosini, *G-invariant persistent homology*, Mathematical Methods in the Applied Sciences, 38, 6 (2015), 1190-1199.

When the functions we consider take values in $\mathbb{R}^n$ instead of $\mathbb{R}$, persistent homology is substituted with *multidimensional persistent homology*:

- A. Cerri, M. Ethier, P. Frosini, *On the geometrical properties of the coherent matching distance in 2D persistent homology*, Journal of Applied and Computational Topology, 3, 4 (2019), 381–422.

Recently, I have started to study the role and use of *group equivariant non-expansive operators (GENEOs)* in geometric deep learning:

- M. G. Bergomi, P. Frosini, D. Giorgi, N. Quercioli, *Towards a topological-geometrical theory of group equivariant non-expansive operators for data analysis and machine learning*, Nature Machine Intelligence, 1, 9 (2 September 2019), 423–433. (Full-text access to a view-only version of this paper is available at the link https://rdcu.be/bP6HV.)

Nowadays, most of my research is devoted to study the natural pseudo-distance d_G, G-invariant persistent homology, multidimensional persistent homology and the topological and geometrical theory of GENEOs.