MA 182, Honors Mathematical Analysis II, Fall '05.
Extra Homework Sheet # 1.

1. For the following converging limits, find an ε–N relationship (that is, for every $\varepsilon > 0$ find a suitable $N \in \mathbb{R}$ that verifies the inequality as in the definition of limit).

   (a) $\lim_{n\to+\infty} \frac{1}{n^3} = 0.$

   (b) $\lim_{n\to+\infty} \frac{2n+1}{n} = 2.$

   (c) $\lim_{n\to+\infty} \frac{n^2}{n^2+1} = 1.$

   (d) $\lim_{n\to+\infty} \frac{1}{2^n} = 0.$ *[Hint: Use that $\forall n \in \mathbb{N}$, $2^n > n$.]*

2. For the following diverging limits, find an R–N relationship (that is, for every $R \in \mathbb{R}$ find a suitable $N \in \mathbb{R}$ that verifies the inequality as in the definition of limit).

   (a) $\lim_{n\to+\infty} (n^2 + n) = +\infty.$

   (b) $\lim_{n\to+\infty} \frac{n^5+n^3}{n^3-1} = +\infty.$ *[Hint: $n^3 - 1 < n^3$.]*

   (c) $\lim_{n\to+\infty} \frac{n^2}{1-n} = -\infty.$ *[Hint: Use a trick similar to problem (b).]*

3. **(Squeeze Theorem for sequences)** Given three sequences $\{a_n\}_{n\in\mathbb{N}}$, $\{b_n\}_{n\in\mathbb{N}}$, $\{c_n\}_{n\in\mathbb{N}}$, such that $\forall n \in \mathbb{N}$, $a_n \leq b_n \leq c_n$ and

$$\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} c_n = L,$$

   then

$$\lim_{n\to+\infty} b_n = L,$$

   [Hint: At some point you want to use a formula like $L - \varepsilon < a_n \leq b_n \leq c_n \leq L + \varepsilon$.]