MA 182, Honors Mathematical Analysis II, Fall '05.
Extra Homework Sheet # 2.

1. Prove that, if $a_n \geq b_n$ for all $n \in \mathbb{N}$, and $b_n \to +\infty$ (as $n \to +\infty$) then $a_n \to +\infty$ (as $n \to +\infty$).

2. Prove that any increasing unbounded (i.e., not bounded) sequence tends to $+\infty$.

3. For the following converging limits, find an ε–δ relationship (that is, for every $\varepsilon > 0$ find a suitable $\delta > 0$ that verifies the inequality as in the definition of limit).

   (a) $\lim_{x \to 1} 3x = 3.$

   (b) $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2} = 4.$

   (c) $\lim_{x \to 0} \left[(x-1)^2 - 2\right] = -1.$

   (d) For $f(x) := \begin{cases} 2 - x & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ -x^2 & \text{for } x > 0 \end{cases}$, $\lim_{x \to 0^+} f(x) = 0.$

   (e) $\lim_{x \to 0^-} \left(\sqrt{-x} + x\right) = 0.$

4. (Do this only if material was covered in class.) For the following diverging limits, find an R–δ relationship (that is, for every $R \in \mathbb{R}$ find a suitable $\delta > 0$ that verifies the inequality as in the definition of limit).

   (a) $\lim_{x \to -2} \dfrac{1}{(x+2)^2} = +\infty.$

   (b) $\lim_{x \to 0^-} \dfrac{1}{x^3 + x} = -\infty.$

   (c) $\lim_{x \to 0^-} \dfrac{1}{x^3 - x} = +\infty.$
   [You may want to be careful here. This is a bit more difficult than the previous problem.]