## MA 281, Honors Mathematical Analysis III, Spring '06. Extra Homework Sheet # 3.

1. Find the interval of convergence of the following power series. Also, comparing them with known power series (via differentiation, integration, or substitution) determine their limiting functions. *[It is best to solve these problems in the given order.]* 

$$\begin{array}{ll} \text{(a)} & \sum_{n=0}^{\infty} x^{3n}. \\ \text{(b)} & \sum_{n=0}^{\infty} \frac{x^{3n}}{27^n}. \\ \text{(c)} & \sum_{n=0}^{\infty} (n+1)x^n. \\ \text{(d)} & \sum_{n=0}^{\infty} (n+1)x^{3n}. \\ \text{(e)} & \sum_{n=0}^{\infty} n(n-1)x^{n-2}. \\ \text{(f)} & \sum_{n=0}^{\infty} n^2x^n. \quad [Hint: \ n^2x^n = x^2[(n^2-n)x^{n-2}] + x[nx^{n-1}]. \ ] \\ \text{(g)} & \sum_{n=1}^{\infty} \frac{x^n}{n}. \quad [Hint: \ Change \ variable \ so \ that \ the \ series \ starts \ at \ 0.] \\ \text{(h)} & \sum_{n=0}^{\infty} \frac{(4x)^n}{n}. \\ \text{(i)} & \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}. \end{array}$$