MA 281, Honors Mathematical Analysis III, Spring '06.
Extra Homework Sheet # 4.

In this homework sheet we are going to use Newton's Second Law of Mechanics,

$$F = mA,$$

where F is the force exherted on a material point of mass m, at a certain instant of time, and A is the resulting acceleration, at that instant.

1. In the following problems, $F(t)$ is the force applied on a point, as a function of time; m is the mass of the point; $V_0 = R'(0)$ is the initial velocity (at time $t = 0$), and $R_0 = R(0)$ is the initial position. Determine the trajectory $R(t)$ of the point.

   (a) $F(t) = (1, t^2)$; $m = 1$; $V_0 = (-1, 2)$; $R_0 = (1, 1)$.

   (b) $F(t) = (\sin t, 0, e^{3t})$; $m = 2$; $V_0 = (0, 0, 0)$; $R_0 = (1, 0, -3)$.

   (c) $F(t) = (1, e^{-t}, 0, (1 + t)^{-3})$; $m = 1/2$; $V_0 = (1, -2, 0, \sqrt{3})$; $R_0 = (0, 0, 0, 0)$.

   (d) $F(t) = (\cos(2t), (2 + t)^{-2}, [2t^2 + 1]e^{t^2})$; $m = 1$; $V_0 = (-1, 0, 1)$; $R_0 = (0, 0, 0)$.

2. A point of mass 1 is moving in $\mathbb{R}^2$, being subject to a constant force field $F = (0, 3)$. Prove that, no matter what the initial position $R(0) = (a, b)$ and the initial velocity $R'(0) = (c, d)$ are, the trajectory is always a parabola.

   [Hint: First determine the trajectory $R(t) = (x(t), y(t))$. Then use the equation of $x(t)$ to find t as a function of x and plug this into $y(t)$.]

3. A material point in the plane is subject to a force field that *depends on the position of the point* in the following way: If the point is at $R = (x, y)$, the force applied to it is $F = F(x, y) = -k(x, y)$, where $k > 0$ is a constant. Therefore, Newton's Second Law in this case reads

   $$-kR(t) = mR''(t).$$

   Prove that, setting $\omega := \sqrt{k/m}$, the curve $R(t) = (2\cos(\omega t - 3), 2\sin(\omega t - 3)$ is a possible trajectory.