MA 281, Honors Mathematical Analysis III, Spring '06.
Some review problems for Midterm # 3.

(NOT A COMPREHENSIVE LIST!)

1. If $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ is the function defined by $f(X) = f(x_1, x_2, \ldots, x_n) = x_k$ (k is a *fixed* integer between 1 and n), **prove** that

$$\lim_{X \to A} f(X) = a_k,$$

where $A = (a_1, a_2, \ldots, a_n) \in \mathbb{R}^n$

[Hint: Use the formulation of limit with ε and δ. Notice that $|x_k - a_k| \leq \|X - A\|$.]

2. Use the above problem to prove that a polynomial $p : \mathbb{R}^n \longrightarrow \mathbb{R}$ is continuous.

[You are allowed to use the result that sums and products of continuous functions are continuous functions.]

3. Guess the following limits (meaning, no proof required):

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1}{(x^2+4y^2)^{3/2}}$

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1}{x^2-y^2}$

(c) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{\arctan(x^3-xy+3z^6)}{x^3-xy+3z^6}$

4. Evaluate the following limits (that is, use either the definition of limit or a theorem or a known result to justify your answer):

(a) $\displaystyle\lim_{(x,y)\to(1,2)} \left[x^2y + \cos(xy)\right]$

(b) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{\sin(x^2+y^2+z^2)^3}{(x^2+y^2+z^2)^3}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{6x^2+y^2}$

(d) $\displaystyle\lim_{(x,y)\to(-1,0)} \frac{e^{(x-1)^2+4y^2}-1}{(x-1)^2+4y^2}$