Esercizi.

Continuità.

Sia $f : A \to \mathbb{R}$, dove $A \subseteq \mathbb{R}^2$. Studiare la continuità di f nel punto (x_0, y_0).

a) $A = \mathbb{R}^2$, $(x_0, y_0) = (1, 2)$ $\qquad\qquad$ $f(x, y) = 3x^2 - \sin(y^2)$ $\qquad\qquad$ [S]

b) $A = \mathbb{R}^2$, $(x_0, y_0) = (0, 0)$ $\qquad\qquad$ $f(x, y) = \sqrt{x^2 + y^2}$ $\qquad\qquad$ [S]

c) $A = \mathbb{R}^2$, $(x_0, y_0) = (0, 0)$ $\qquad\qquad$ $f(x, y) = \begin{cases} \dfrac{xy^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ $\qquad$ [S]

d) $A = \mathbb{R}^2$, $(x_0, y_0) = (0, 0)$ $\qquad\qquad$ $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ $\qquad$ [N]

e) $A = \left\{ (x, y) \in \mathbb{R}^2 \, : \, x \leq y \leq 4x \right\}$, $(x_0, y_0) = (0, 0)$ $\qquad$ $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ $\qquad$ [N]

f) $A = \left\{ (x, y) \in \mathbb{R}^2 \, : \, 0 \leq y \leq x^2 \right\}$, $(x_0, y_0) = (0, 0)$ $\qquad$ $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ $\qquad$ [S]

g) $A = \mathbb{R}^2$, $(x_0, y_0) = (1, 1)$ $\qquad\qquad$ $f(x, y) = \begin{cases} 1, & x \leq y^2 \\ 0, & x > y^2 \end{cases}$ $\qquad$ [N]

h) $A = \mathbb{R}^2$, $(x_0, y_0) = (0, 0)$ $\qquad\qquad$ $f(x, y) = \begin{cases} y \log(x^2 + y^2), & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ $\qquad$ [S]

i) $A = \left\{ (x, y) \in \mathbb{R}^2 \, : \, y > -1 \right\}$, $(x_0, y_0) = (0, 0)$ $\qquad$ $f(x, y) = \begin{cases} 3 \dfrac{\sin(y) - \log(1 + y)}{e^{y^2} - \cos(y)}, & y \neq 0 \\ 1, & y = 0 \end{cases}$ $\qquad$ [S]

l) $A = \mathbb{R}^2$, $(x_0, y_0) = (0, 0)$ $\qquad\qquad$ $f(x, y) = \begin{cases} \dfrac{y \sqrt{|x|}}{|x| + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ $\qquad$ [N]

m) $A = \left\{ (x, y) \in \mathbb{R}^2 \, : \, 0 \leq y \leq x \right\}$, $(x_0, y_0) = (0, 0)$ $\qquad$ $f(x, y) = \begin{cases} \dfrac{y \sqrt{|x|}}{|x| + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ $\qquad$ [S]