Esercizi.

Differenziabilità.

Sia $f : A \to \mathbb{R}$, dove $A \subseteq \mathbb{R}^2$. Studiare la continuità, l'esistenza di derivate direzionali e la differenziabilità, nel punto (x_0, y_0).

a) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \sqrt{x^2 + 4y^2}$ $\quad [C, NoDe]$

b) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \sqrt{|xy|}$ $\quad [C, NoDe]$

c) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ $\quad [NoC, NoDe]$

d) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \begin{cases} \dfrac{xy^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ $\quad [C, De, NoDi]$

e) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ $\quad [Di]$

f) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \sqrt{|x|^{\frac{5}{2}} + |y|^{\frac{7}{3}}}$ $\quad [Di]$

g) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,1)$ $\quad f(x,y) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) + y, & x \neq 0 \\ y, & x = 0 \end{cases}$ $\quad [Di]$

h) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \begin{cases} e^{-x^2} \sin(y) \dfrac{x^2y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ $\quad [Di]$

i) $A = \mathbb{R}^2,\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \cos(\sqrt{x^2 + y^2})$ $\quad [Di]$

l) $A = \left\{(x,y) \in \mathbb{R}^2 \,:\, x^2 + y^2 < 1\right\},\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \begin{cases} \dfrac{xy}{\log\left(1 - x^2 - y^2\right)}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ $\quad [NoC, NoDe]$

m) $A = \left\{(x,y) \in \mathbb{R}^2 \,:\, x^2 + y^2 < 1\right\},\ (x_0, y_0) = (0,0)$ $\quad f(x,y) = \begin{cases} \dfrac{xy(x^2+y^2)}{\log\left(1 - x^2 - y^2\right)}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ $\quad [Di]$