Esercizi.

Insiemi, topologia.

Rappresentare graficamente i seguenti insiemi. Inoltre studiare: l'interno, il derivato, la frontiera, la chiusura, la limitatezza. Infine, stabilire quali sono aperti, chiusi, compatti.

a) $A \subseteq \mathbb{R}^2,\ A = \mathbb{R}^2;\quad B \subseteq \mathbb{R}^2,\ B = \mathbb{Z}^2;\quad C \subseteq \mathbb{R}^2,\ C = \mathbb{Q}^2$

b) $A = \{(x,y) \in \mathbb{R}^2 : x = 0\};\quad B = \{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Z},\ y = 3\};\quad C = \{(x,y) \in \mathbb{R}^2 : x = 5,\ y \in \mathbb{Q}\};$

c) $A = \{(x,y) \in \mathbb{R}^2 : 2x + 5y - 3 \geq 3x - 7y + 4\}$

d) $A = \{(x,y) \in \mathbb{R}^2 : x > |y - 1|\}$

e) $A = \{(x,y) \in \mathbb{R}^2 : |x| \leq |y - 1|\}$

f) $A = \{(x,y) \in \mathbb{R}^2 : -1 \leq 2x + 3y \leq 4,\ 5 \leq x - y \leq 7\}$

g) $A = \{(x,y) \in \mathbb{R}^2 : (x + 3)^2 + y^2 \geq 4\}$

h) $A = \{(x,y) \in \mathbb{R}^2 : x^2 - (y - 1)^2 < 9\}$

i) $A = \{(x,y) \in \mathbb{R}^2 : 3x \leq 4y^2\}$

j) $A = \{(x,y) \in \mathbb{R}^2 : 4 < x^2 + 4y^2 \leq 9,\ |y| \leq x\}$

k) $A = \{(x,y) \in \mathbb{R}^2 : x^2 \leq y + 1 < 5,\ x \leq -3\}$

l) $A \subseteq \mathbb{R}^3,\ A = \mathbb{R}^3;\quad B \subseteq \mathbb{R}^3,\ B = \mathbb{Z}^3;\quad C \subseteq \mathbb{R}^3,\ C = \mathbb{Q}^3$

m) $A \subseteq \mathbb{R}^3,\ A = \mathbb{R}^2 \times \{0\};\quad B \subseteq \mathbb{R}^3,\ B = \mathbb{Z}^2 \times \{3\};\quad C \subseteq \mathbb{R}^3,\ C = \{1\} \times \mathbb{Q}^2$

n) $A = \{(x,y,z) \in \mathbb{R}^3 : (x - 1)^2 + (y - 2)^2 + (z - 3)^2 \leq 4\}$

o) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + 4y^2 + z^2 = 4,\ x < 0\}$

p) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 - z^2 < 1\}$

q) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 - y^2 - z^2 > 1\}$

r) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = z\}$

s) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 - y^2 \leq z\}$

t) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = z^2\}$

u) $A = \{(x,y,z) \in \mathbb{R}^3 : (x - 3)^2 + y^2 \leq 4\}$

v) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + (y + 1)^2 \leq 9,\ |z| < 1\}$

w) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4,\ x^2 + z^2 > y^2\}$

x) $A = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 > 4,\ -2 < z \leq x^2 + y^2\}$

y) $A = \{(x,y,z) \in \mathbb{R}^3 : |x - 1| < 2,\ |y| \geq 1\}$

z) $A = \{(x,y,z) \in \mathbb{R}^3 : xy < 1,\ |z - 2| < 1\}$