

Esercizi.

Integrali multipli

Calcolare l'integrale delle seguenti funzioni sugli insiemi indicati:

- a)  $f(x, y) = 5x + 1$ ,  $A = \{(x, y) \in \mathbb{R}^2 : x^2 \leq y + 1 \leq 8, x \leq -1\}$
- b)  $f(x, y) = 3x$ ,  $A = \{(x, y) \in \mathbb{R}^2 : 2y^2 \leq x \leq y^2 + 4\}$
- c)  $f(x, y) = 1$ ,  $A = \{(x, y) \in \mathbb{R}^2 : x^2 + 4y^2 \leq 4, |y| \leq x\}$
- d)  $f(x, y) = 1$ ,  $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 9, y^2 \geq -x\}$
- e)  $f(x, y) = e^{-y^2}$ ,  $A = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq y \leq 1\}$
- f)  $f(x, y) = 2x + 3y$ ,  $A = \{(x, y) \in \mathbb{R}^2 : -1 \leq 2x + 3y \leq 4, 5 \leq x - y \leq 7\}$
- g)  $f(x, y) = e^{3x+7y}$ ,  $A = \{(x, y) \in \mathbb{R}^2 : (3x + 7y)^2 \leq x - 2y \leq 8\}$
- h)  $f(x, y) = 2x + y$ ,  $A = \{(x, y) \in \mathbb{R}^2 : 4 \leq x^2 + y^2 \leq 9, |y| \leq x\}$
- i)  $f(x, y) = 1$ ,  $A = \{(x, y) \in \mathbb{R}^2 : 9x^2 + 4y^2 \leq 1, \sqrt{3}x \leq 2y\}$
- l)  $f(x, y, z) = 1$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + z^2 \leq y^2 + 1 \leq 2\}$
- m)  $f(x, y, z) = 1$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 9, y^2 + z^2 \leq x\}$
- n)  $f(x, y, z) = y$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4, x^2 + z^2 \leq y^2\}$
- o)  $f(x, y, z) = 1$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4, x \geq 1\}$
- p)  $f(x, y, z) = y^2$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4\}$
- q)  $f(x, y, z) = x^2 z$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 9, |z| \leq 1\}$
- r)  $f(x, y, z) = 1$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 4, -3 \leq z \leq x^2 + y^2\}$
- s)  $f(x, y, z) = x$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : (x - 1)^2 + (y - 2)^2 + (z - 3)^2 \leq 4\}$
- t)  $f(x, y, z) = 1$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4, x^2 + y^2 \geq z^2 + 2, z \leq 0\}$
- u)  $f(x, y, z) = z^2$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 4, x^2 + y^2 \geq z^2 + 2\}$
- v)  $f(x, y, z) = 1$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1, a, b, c > 0\}$
- z)  $f(x, y, z) = 1$ ,  $A = \{(x, y, z) \in \mathbb{R}^3 : \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} \leq 1, |z| \leq 1, a, b, c > 0\}$