Esercizi.

Limiti.

Sia $f : A \to \mathbb{R}$, dove $A \subseteq \mathbb{R}^2$. Determinare il dominio A e studiare i seguenti limiti

$a)\ f(x,y) = 2x^2 + 3y^3 \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2; 0]$

$b)\ f(x,y) = \sqrt{2x^2 + 3y^4} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2; 0]$

$c)\ f(x,y) = \dfrac{xy}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2 \setminus \{(0,0)\}; \nexists]$

$d)\ f(x,y) = \dfrac{xy}{\sqrt{x^2+y^2}} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2 \setminus \{(0,0)\}; 0]$

$e)\ f(x,y) = \dfrac{2xy}{x^2+xy+y^2} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2 \setminus \{(0,0)\}; \nexists]$

$f)\ f(x,y) = \dfrac{2x^2y-5y^2}{x^2-2xy+y^2} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \big\{(x,y) \in \mathbb{R}^2 \,:\, x \neq y\big\}; \nexists]$

$g)\ f(x,y) = \dfrac{3xy}{2x^4+5y^6} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2 \setminus \{(0,0)\}; \nexists]$

$h)\ f(x,y) = \dfrac{4xy^2}{\sqrt{2x^6+y^6}} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2 \setminus \{(0,0)\}; \nexists]$

$i)\ f(x,y) = \dfrac{3x^2\sin(y)}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2 \setminus \{(0,0)\}; 0]$

$l)\ f(x,y) = \dfrac{\sin\left(x^2+y^2\right)}{2xy} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \big\{(x,y) \in \mathbb{R}^2 \,:\, x \neq 0, y \neq 0\big\}; \nexists]$

$m)\ f(x,y) = x\log(y) \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \big\{(x,y) \in \mathbb{R}^2 \,:\, y > 0\big\}; \nexists]$

$n)\ f(x,y) = \dfrac{x}{y}\sqrt{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \big\{(x,y) \in \mathbb{R}^2 \,:\, y \neq 0\big\}; \nexists]$

$o)\ f(x,y) = \dfrac{\sin(x^2+y^2)}{x^2+y^2} \qquad \lim_{(x,y)\to(0,0)} f(x,y), \qquad [A = \mathbb{R}^2 \setminus \{(0,0)\}; 1]$

$p)$ $f(x,y) = 2x^2 y \log(x^4 + y^2)$ $\lim_{(x,y)\to(0,0)} f(x,y)$, $[A = \mathbb{R}^2 \setminus \{(0,0)\}; 0]$

$q)$ $f(x,y) = \dfrac{x}{y}$ $\lim_{(x,y)\to\infty} f(x,y)$, $[A = \{(x,y) \in \mathbb{R}^2 \,:\, y \neq 0\}; \nexists]$

$r)$ $f(x,y) = x^2 e^{-y^2}$ $\lim_{(x,y)\to\infty} f(x,y)$, $[A = \mathbb{R}^2; \nexists]$

$s)$ $f(x,y) = (3 + \cos(xy)) \log(x^2 + y^2)$ $\lim_{(x,y)\to\infty} f(x,y)$, $[A = \mathbb{R}^2 \setminus \{(0,0)\}; +\infty]$

$t)$ $f(x,y) = \sqrt{x^2 + y^2}\, e^{-(x^2+y^2)}$ $\lim_{(x,y)\to\infty} f(x,y)$, $[A = \mathbb{R}^2; 0]$

$u)$ $f(x,y) = \dfrac{2xy^2}{x^4 + y^4}$ $\lim_{(x,y)\to\infty} f(x,y)$, $[A = \mathbb{R}^2 \setminus \{(0,0)\}; 0]$

$v)$ $f(x,y) = \dfrac{x^4 + y^2}{x^2 y}$ $\lim_{(x,y)\to\infty} f(x,y)$, $[A = \{(x,y) \in \mathbb{R}^2 \,:\, x \neq 0, y \neq 0\}; \nexists]$

Sia $f : A \to \mathbb{R}$, dove $A \subseteq \mathbb{R}^3$. Determinare il dominio A e studiare i seguenti limiti

$a)$ $f(x,y,z) = 2x^2 + 3y^3 + 4z^4$ $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$, $[A = \mathbb{R}^3; 0]$

$b)$ $f(x,y,z) = \dfrac{3xyz}{x^2 + y^2 + z^2}$ $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$, $[A = \mathbb{R}^3 \setminus \{(0,0,0)\}; 0]$

$c)$ $f(x,y,z) = \dfrac{x}{y^2 + z^2}$ $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$, $[A = \{(x,y,z) \in \mathbb{R}^3 \,:\, (y,z) \neq (0,0)\}; \nexists]$

$d)$ $f(x,y,z) = y \log(x^2 + z^2)$ $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$, $[A = \{(x,y,z) \in \mathbb{R}^3 \,:\, (x,z) \neq (0,0)\}; \nexists]$

$e)$ $f(x,y,z) = \dfrac{x \sin(y)}{\sqrt{x^2 + y^2 + z^2}}$ $\lim_{(x,y,z)\to(0,0,0)} f(x,y,z)$, $[A = \mathbb{R}^3 \setminus \{(0,0,0)\}; 0]$

$f)$ $f(x,y,z) = \dfrac{x}{x^2 + 3y^2 + 4z^2}$ $\lim_{(x,y,z)\to\infty} f(x,y,z)$, $[A = \mathbb{R}^3 \setminus \{(0,0,0)\}; 0]$

$g)$ $f(x,y,z) = \dfrac{xyz^2}{x^4 + y^4 + z^4}$ $\lim_{(x,y,z)\to\infty} f(x,y,z)$, $[A = \mathbb{R}^3 \setminus \{(0,0,0)\}; \nexists]$