Esercizi.

Limiti con l'ausilio del calcolo differenziale.

Scrivere la formula di Taylor con resto di Peano, in $x_0 = 0$ delle seguenti funzioni

a) $f(x) = e^x$ *b)* $f(x) = \sin x$ *c)* $f(x) = \cos x$ *d)* $f(x) = \tan x$

e) $f(x) = \arctan x$ *f)* $f(x) = \log(1+x)$ *g)* $f(x) = \sinh x$ *h)* $f(x) = \cosh x$

i) $f(x) = \tanh x$ *l)* $f(x) = \dfrac{1}{1-x}$ *m)* $f(x) = \arcsin x$ *n)* $f(x) = (1+x)^a\ ,\ a > 0$

Calcolare i seguenti limiti

a) $\displaystyle\lim_{x\to 1} \frac{\sqrt{x}-\sqrt[5]{x}}{2x-3}$ *b)* $\displaystyle\lim_{x\to 0} \frac{x+\sin x}{x-\sin x}$ *c)* $\displaystyle\lim_{x\to +\infty} \frac{x+\sin x}{x-\sin x}$

d) $\displaystyle\lim_{x\to +\infty} \frac{e^x}{x^a}\ ,\ a > 0$ *e)* $\displaystyle\lim_{x\to 0^+} x^x$ *f)* $\displaystyle\lim_{x\to 1} x^{\frac{1}{x-1}}$

g) $\displaystyle\lim_{x\to e} \frac{\sqrt{3x}-\sqrt{3e}}{\log(x)-1}$ *h)* $\displaystyle\lim_{x\to +\infty} \frac{\log^b(x)}{x^a}\ ,\ a,b > 0$ *i)* $\displaystyle\lim_{x\to 0} \frac{x\cos x-\sin x}{x^3}$

l) $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\tan(3x)}{\tan(x)}$ *m)* $\displaystyle\lim_{x\to 0} \frac{x-\sinh(x)}{x-\sin(x)}$ *n)* $\displaystyle\lim_{x\to 0^+} \frac{\log(\sin(ax))}{\log(\sin(bx))}\ ,\ a,b > 0$

o) $\displaystyle\lim_{x\to +\infty} x - x^2\log(1+\frac{1}{x})$ *p)* $\displaystyle\lim_{x\to +\infty} \frac{\log(2+\cos x)}{\log(2+\sin x)}$ *q)* $\displaystyle\lim_{x\to 0} \frac{\log(\cos x)}{2x^2}$

r) $\displaystyle\lim_{x\to 0} \frac{1}{x}-\frac{1}{\sin x}$ *s)* $\displaystyle\lim_{x\to 0} \frac{1-\sqrt{1+x^2}\cos x}{\tan^4 x}$ *t)* $\displaystyle\lim_{x\to 0} \frac{\sin\left(x-\sin x\right)}{1-\sqrt{1+x^3}}$

u) $\displaystyle\lim_{x\to 0} \frac{e^{-\frac{x^2}{2}}-\cos x}{2x^4}$ *v)* $\displaystyle\lim_{x\to 0} \frac{x(6+x^2)-6\sinh x}{x^5}$ *z)* $\displaystyle\lim_{x\to 0} \frac{x(1+x)-e^x\sin x}{x^3}$