

Esercizi.  
Numeri complessi

Scrivere in forma algebrica i seguenti numeri complessi

$$a) z = i(1+i) \quad [sol : -1+i];$$

$$b) z = \frac{(i+1)^2 + (1+2i)^2}{(1-2i)} \quad [sol : -3];$$

$$c) z = \frac{1-i}{1+i} \quad [sol : -i];$$

$$d) z = \frac{1-3i}{1+i} \quad [sol : -1-2i];$$

$$e) z = \frac{i(i-1)}{(1+i)^2} \quad [sol : -\frac{1}{2} + \frac{1}{2}i];$$

$$f) z = (1+i)^{10} \quad [sol : 32i];$$

$$g) z = \frac{3}{(-1 + \frac{i}{\sqrt{3}})^4} \quad [sol : -\frac{27}{32} + i\frac{27}{32}\sqrt{3}];$$

$$h) z = e^{(2+i)^3} \quad [sol : e^2 \cos 11 + ie^2 \sin 11];$$

Scrivere in forma trigonometrica ed esponenziale i seguenti numeri complessi

$$a) z = 1+i \quad [sol : \sqrt{2}(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})) = \sqrt{2}e^{\frac{\pi}{4}i}];$$

$$b) z = 1-i\sqrt{3} \quad [sol : 2(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})) = 2e^{-\frac{\pi}{3}i}];$$

$$c) z = 2+2i\sqrt{3} \quad [sol : 4(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})) = 4e^{\frac{\pi}{3}i}];$$

$$d) z = -1 - \frac{i}{\sqrt{3}} \quad [sol : \frac{2\sqrt{3}}{3}(\cos(\frac{7\pi}{6}) + i\sin(\frac{7\pi}{6})) = \frac{2\sqrt{3}}{3}e^{\frac{7\pi}{6}i}];$$

$$e) z = \frac{1}{3+3i} \quad [sol : \frac{1}{3\sqrt{2}}(\cos(\frac{-\pi}{4}) + i\sin(\frac{-\pi}{4})) = \frac{1}{3\sqrt{2}}e^{-\frac{\pi}{4}i}];$$

$$f) z = \sin \theta + i \cos \theta \quad [sol : \cos(\frac{\pi}{2} - \theta) + i\sin(\frac{\pi}{2} - \theta) = e^{(\frac{\pi}{2} - \theta)i}];$$

Calcolare le seguenti radici  $n$ -esime e rappresentarle sul piano complesso

a)  $z^2 = 2i$  [sol :  $z_0 = \sqrt{2}e^{\frac{\pi}{4}i}$ ,  $z_1 = \sqrt{2}e^{\frac{5\pi}{4}i}$ ];

b)  $z^3 = -1$  [sol :  $z_0 = e^{\frac{\pi}{3}i}$ ,  $z_1 = -1$ ,  $z_2 = e^{\frac{5\pi}{3}i}$ ];

c)  $z^4 = -1 + i\sqrt{3}$  [sol :  $z_0 = \sqrt[4]{2}e^{\frac{\pi}{6}i}$ ,  $z_1 = \sqrt[4]{2}e^{\frac{2\pi}{3}i}$ ,  $z_2 = \sqrt[4]{2}e^{\frac{7\pi}{6}i}$ ,  $z_3 = \sqrt[4]{2}e^{\frac{5\pi}{3}i}$ ];

d)  $z^4 = 16i$  [sol :  $z_0 = 2e^{\frac{\pi}{8}i}$ ,  $z_1 = 2e^{\frac{5\pi}{8}i}$ ,  $z_2 = 2e^{\frac{9\pi}{8}i}$ ,  $z_3 = 2e^{\frac{13\pi}{8}i}$ ];

e)  $z^5 = i$  [sol :  $z_0 = e^{\frac{\pi}{10}i}$ ,  $z_1 = e^{\frac{5\pi}{10}i}$ ,  $z_2 = e^{\frac{9\pi}{10}i}$ ,  $z_3 = e^{\frac{13\pi}{10}i}$ ,  $z_4 = e^{\frac{17\pi}{10}i}$ ];

f)  $z^6 = -8$  [sol :  $z_0 = \sqrt{2}e^{\frac{\pi}{6}i}$ ,  $z_1 = \sqrt{2}e^{\frac{\pi}{2}i}$ ,  $z_2 = \sqrt{2}e^{\frac{5\pi}{6}i}$ ,  $z_3 = \sqrt{2}e^{\frac{7\pi}{6}i}$ ,  $z_4 = \sqrt{2}e^{\frac{3\pi}{2}i}$ ,  $z_5 = \sqrt{2}e^{\frac{11\pi}{6}i}$ ];

Risolvere le seguenti equazioni in campo complesso

a)  $z^4 + 3z^2 + 2 = 0$  [sol :  $\pm i, \pm i\sqrt{2}$ ];

b)  $z^3 - z^2 + z = 0$  [sol :  $0, \frac{1}{2}(1 \pm i\sqrt{3})$ ];

c)  $z^2 - (2+i)z + 3i - 3 = 0$  [sol :  $3, i - 1$ ];

d)  $z^3 - |z|^2 = 0$  [sol :  $0, 1, \frac{1}{2}(-1 \pm i\sqrt{3})$ ];

e)  $|z|^2 + 5z + 10i = 0$  [sol :  $-1 - 2i, -4 - 2i$ ];

f)  $z^3 - 2\bar{z} = 0$  [sol :  $0, \pm\sqrt{2}, \pm i\sqrt{2}$ ];

g)  $\operatorname{Im}\left(\frac{1}{z}\right) = -1$  [sol :  $|z - \frac{i}{2}| = \frac{1}{2}$ ];