Esercizi.

Successioni

Verificare in base alla definizione di limite:

$a)\ \lim_{n\to\infty} \frac{1}{n} = 0$ $b)\ \lim_{n\to\infty} -n = -\infty$ $c)\ \lim_{n\to\infty} n^2 = +\infty$

$d)\ \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ $e)\ \lim_{n\to\infty} \frac{n+1}{n+2} = 1$ $f)\ \lim_{n\to\infty} \frac{n^2+1}{n^2+n} = 1$

$g)\ \lim_{n\to\infty} \frac{n+1}{n^2+1} = 0$ $h)\ \lim_{n\to\infty} \frac{n^2+1}{n} = +\infty$ $i)\ \lim_{n\to\infty} \frac{1-n^2}{n} = -\infty$

Sia a_n una successione, dimostrare che:

$a)\ o(a_n) + o(a_n) = o(a_n),\ n \to \infty$ $b)\ o(a_n) - o(a_n) = o(a_n),\ n \to \infty$

$c)\ o(c\, a_n) = o(a_n),\ n \to \infty,\ \forall\, c \in \mathbb{R} \setminus \{0\}$ $d)\ c\, o(a_n) = o(a_n),\ n \to \infty,\ \forall\, c \in \mathbb{R} \setminus \{0\}$

$e)\ \big(o(a_n)\big)^p = o(a_n^p),\ n \to \infty,\ \forall\, p > 0$ $f)\ a_n^p\, o(a_n) = o(a_n^{p+1}),\ n \to \infty$

$g)\ o\big(o(a_n)\big) = o(a_n),\ n \to \infty$ $h)\ o\big(a_n + o(a_n)\big) = o(a_n),\ n \to \infty$

Studiare il comportamento delle seguenti successioni a_n, per $n \to \infty$

$a)\ a_n = n^p,\ p \in \mathbb{Z}$ $b)\ a_n = c^n,\ c \in \mathbb{R}$ $c)\ a_n = \sqrt[n]{n^p},\ p \in \mathbb{Z}$ $d)\ a_n = (1+n^{-2})^n,$

$e)\ a_n = \left(\frac{2n+4}{n+3}\right)^n,$ $f)\ a_n = \left(\frac{n+4}{n+3}\right)^n,$ $g)\ a_n = \frac{n^p}{c^n},\ p > 0,\ c > 1$ $h)\ a_n = \frac{c^n}{n!},\ c > 1$

$i)\ a_n = \frac{n!}{n^n},$ $l)\ a_n = \frac{5n^3+4}{2^n},$ $m)\ a_n = \frac{2^n+n!}{3^n+n^5},$ $n)\ a_n = \frac{e^n-2^n}{e^n+2^n},$

$o)\ a_n = \frac{\sin(n)}{n},$ $p)\ a_n = (-1)^n \frac{2^n}{n!+n^5},$ $q)\ a_n = \frac{\sqrt[n]{n!}}{n},$ $r)\ a_n = \frac{n^3+n\sin(n)}{n^3\arctan(n)+n^2}$