Integral formulas for a class of curvature PDE's and applications to isoperimetric inequalities and to symmetry problems
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Corrigendum

Vittorio Martino(1) & Annamaria Montanari(2)

In the proof of Theorem 1.2 (Isoperimetric estimates) in [2], we used inequality (15) without actually knowing the sign of the eigenvalues of $\partial\bar{\partial}f$, where f is a solution of (14); we thank Professor Zbigniew Błocki for having pointed out it to us.
So, Theorem 1.2 in [2] is true for $j = 1$, since in that case the inequality (15) holds without any hypothesis on the sign of the eigenvalues of the matrix A (the case $j = 1$ in the statement of Theorem 1.2 corresponds to $j = 2$ in formula (15)).
However, here we show that once one has the case $j = 1$, by using some Gärding inequalities, one can prove the formula (2) in Theorem 1.2 also for $j > 1$.
We use the same notation as in [2]. Let $K_{\partial\Omega} = K^{(1)}_{\partial\Omega}$, then we have the following

Theorem (Isoperimetric estimate). *Let Ω be a bounded domain of $\mathbb{C}^{n+1}$ with boundary a real hypersurface of class C^∞. If $K_{\partial\Omega}$ is positive then*

$$\int_{\partial\Omega} \frac{1}{K_{\partial\Omega}(x)} d\sigma(x) \geq 2(n+1)|\Omega| \tag{1}$$

where $|\Omega|$ is the Lebesgue measure of Ω. If $K_{\partial\Omega}$ is constant, then the equality holds in (1) if and only if Ω is a ball of radius $\frac{1}{K_{\partial\Omega}}$.

Proof. As in [2], Theorem 1.2, case $j = 1$. □

Here we recall some inequalities from [1]. Let $\lambda(A) = \{\lambda_1, \dots, \lambda_n\}$ the vector of eigenvalues of a $n \times n$ Hermitian Matrix A. For $k \in \{1, \dots, n\}$ we define the normalized $k-$th elementary symmetric function of the eigenvalues of A as $s_k(A) = \frac{\sigma_k(A)}{\binom{n}{k}}$. We also denote

[1]Dipartimento di Matematica, Università di Bologna, piazza di Porta S.Donato 5, 40126 Bologna, Italy. E-mail address: vittorio.martino3@unibo.it

[2]Dipartimento di Matematica, Università di Bologna, piazza di Porta S.Donato 5, 40126 Bologna, Italy. E-mail address: annamaria.montanari@unibo.it

by $\Gamma_k \subseteq \mathbb{R}^n$ the connected component of the set $\{\lambda \in \mathbb{R}^n; s_k(A) > 0\}$ which contains the vector $(1, \dots, 1)$. We have that $\Gamma_k \subseteq \Gamma_i$, if $i \le k$ and moreover if $\lambda(A) \in \Gamma_k$ then it holds

$$(s_k(A))^{1/k} \le s_1(A). \tag{2}$$

Now, we have the following

Corollary. *Let $j \in \{2, \dots, n\}$. Let Ω be a bounded domain of $\mathbb{C}^{n+1}$ with boundary a real hypersurface of class C^∞. If $K^{(j)}_{\partial\Omega}$ is positive then*

$$\int_{\partial\Omega} \left(\frac{1}{K^{(j)}_{\partial\Omega}(x)} \right)^{1/j} d\sigma(x) \ge 2(n+1)|\Omega| \tag{3}$$

where $|\Omega|$ is the Lebesgue measure of Ω. If $K^{(j)}_{\partial\Omega}$ is constant, then the equality holds in (3) *if and only if Ω is a ball of radius $\left(\frac{1}{K^{(j)}_{\partial\Omega}} \right)^{1/j}$.*

Proof. Since $\partial\Omega$ is compact, there exists at least a point of ellipticity, namely there exists $p_0 \in \partial\Omega$ such that all the eigenvalues of the second fundamental form (at p_0) are strictly positive; therefore all the eigenvalues of L_{p_0} (the Levi form at p_0) are strictly positive: in particular $\lambda(L_{p_0}) \in \Gamma_j$. Now, since the function $K^{(j)}_{\partial\Omega} := s_j(L_p)$ is positive and continuous on $\partial\Omega$, we have that $\lambda(L_p) \in \Gamma_j$, $\forall p \in \partial\Omega$. Hence, by (1) and (2), we obtain

$$\int_{\partial\Omega} \left(\frac{1}{K^{(j)}_{\partial\Omega}(x)} \right)^{1/j} d\sigma(x) \ge \int_{\partial\Omega} \frac{1}{K_{\partial\Omega}(x)} d\sigma(x) \ge 2(n+1)|\Omega|.$$

□

References

[1] L.Gärding, An inequality for hyperbolic polynomials. J. Math. Mech. 8 1959 957-965.

[2] V.Martino, A.Montanari, Integral formulas for a class of curvature PDE's and applications to isoperimetric inequalities and to symmetry problems Forum Mathematicum, vol. 22, 2010, 255-267