DERIVATE
FUNZIONI
COMPOSTE

$y = [f(x)]^{\alpha} \qquad y' = \alpha \cdot [f(x)]^{\alpha - 1} \cdot f'(x)$

$y = (x - x^2)^3 \qquad y' = 3(x - x^2)^2 \cdot (1 - 2x)$

$y = \frac{(1-x)^4}{2} \qquad y = \frac{1}{2}(1-x)^4 \qquad y' = \frac{1}{\cancel{2}} \cdot \overset{2}{\cancel{4}}(1-x)^3(-1)$

$y' = -2(1-x)^3$

$y = \sqrt[3]{(1-x^2)} \qquad y = (1-x^2)^{\frac{1}{3}} \qquad y' = \frac{1}{3}(1-x^2)^{\frac{1}{3}-1} \cdot (-2x)$

$y' = \frac{1}{3}(1-x^2)^{-\frac{2}{3}}(-2x)$

$y' = \frac{-2x}{3 \cdot \sqrt[3]{1-x^2}}$

$y = \frac{2}{\sqrt{4-x}} \qquad y = 2(4-x)^{-\frac{1}{2}} \qquad y' = \cancel{2} \cdot \left(-\frac{1}{\cancel{2}}\right) \cdot (4-x)^{-\frac{1}{2}-1}(-1) =$

$= +(4-x)^{-\frac{3}{2}} = \frac{1}{\sqrt{(4-x)^3}}$

$y = \frac{1}{2-3x} \qquad y = (2-3x)^{-1} \qquad y' = -1(2-3x)^{-2}(-3) =$

$= +\frac{3}{(2-3x)^2}$

$y = \frac{4}{(1-x)^2} \qquad y = 4 \cdot (1-x)^{-2} \qquad y' = 4 \cdot (-2)(1-x)^{-3} \cdot (-1)$

$y' = +\frac{8}{(1-x)^3}$

$y = \sqrt[4]{(1-x)^3} \Rightarrow y = (1-x)^{\frac{3}{4}} \Rightarrow y' = \frac{3}{4}(1-x)^{\frac{3}{4}-1}(-1)$

$= \frac{3}{4}(1-x)^{-\frac{1}{4}}(-1)$

$= -\frac{3}{4} \cdot \frac{1}{\sqrt[4]{(1-x)}}$

$y = \ln^2 x \qquad y' = 2\ln x \cdot \frac{1}{x} = \frac{2}{x}\ln x$

$y = \frac{\ln^5 x}{2} \qquad y = \frac{1}{2}\ln^5 x \qquad y' = \frac{1}{2}\cdot 5\ln^4 x \cdot \frac{1}{x} = \frac{5}{2x}\ln^4 x$

$y = \sqrt{\ln x} \qquad y = (\ln x)^{\frac{1}{2}} \qquad y' = \frac{1}{2}(\ln x)^{\frac{1}{2}-1}\cdot\frac{1}{x} = \frac{1}{2}(\ln x)^{-\frac{1}{2}}\cdot\frac{1}{x}$

$= \frac{1}{2x\sqrt{\ln x}}$

$y = \frac{2}{3\cdot\ln x} \qquad y = \frac{2}{3}(\ln x)^{-1} \qquad y' = \frac{2}{3}\cdot(-1)(\ln x)^{-2}\frac{1}{x} = -\frac{2}{3x}\cdot\frac{1}{\ln^2 x}$

$y = \frac{\operatorname{sen}^3 x}{2} \qquad y = \frac{1}{2}\operatorname{sen}^3 x \qquad y' = \frac{1}{2}\cdot 3\operatorname{sen}^2 x\cdot\cos x$

$y = \sqrt[3]{\operatorname{sen} x} \qquad y = \operatorname{sen}^{\frac{1}{3}} x \qquad y' = \frac{1}{3}(\operatorname{sen} x)^{\frac{1}{3}-1}(\cos x) = \frac{1}{3}(\operatorname{sen} x)^{-\frac{2}{3}}\cos x$

$= \frac{\cos x}{3\sqrt[3]{\cos^2 x}}$

$y = \frac{3}{\operatorname{sen}^2 x} \qquad y = 3\cdot(\operatorname{sen} x)^{-2} \qquad y' = 3\cdot(-2)(\operatorname{sen} x)^{-3}(\cos x) =$

$= \frac{-6\cos x}{\operatorname{sen}^3 x}$

$y = \frac{\cos^4 x}{2} \qquad y = \frac{1}{2}\cos^4 x \qquad y' = \frac{1}{2}\cdot 4\cos^3 x(-\operatorname{sen} x) =$

$= -2\cos^3 x\operatorname{sen} x$

$y = \frac{2}{\cos x} \qquad y = 2\cdot(\cos x)^{-1} \qquad y' = 2\cdot(-1)(\cos x)^{-2}(-\operatorname{sen} x)$

$= \frac{2\operatorname{sen} x}{\cos^2 x}$

$y = \frac{\sqrt{\cos x}}{3} \qquad y = \frac{1}{3}(\cos x)^{\frac{1}{2}} \qquad y' = \frac{1}{3}\cdot\frac{1}{2}\cdot(\cos x)^{\frac{1}{2}-1}(-\operatorname{sen} x)$

$= -\frac{1}{6}\operatorname{sen} x(\cos x)^{-\frac{1}{2}}$

$= \frac{-\operatorname{sen} x}{6\sqrt{\cos x}}$

$y = \log_a f(x) \qquad y' = \frac{1}{f(x) \cdot \ln a} \cdot f'(x)$

$y = \ln f(x) \qquad y' = \frac{1}{f(x)} \circ f'(x)$

$$y = \ln(1-x^2) \qquad y' = \frac{1}{1-x^2} \cdot (-2x) = \frac{-2x}{1-x^2}$$

$$y = \frac{\ln(1-x)}{3} \qquad y = \frac{1}{3}\ln(1-x) \qquad y' = \frac{1}{3} \cdot \frac{1}{1-x} \cdot (-1) = \frac{-1}{3(1-x)}$$

$$y = \frac{1}{2} \ln^3(x^4) \qquad y' = \frac{1}{\cancel{2}} \cdot 3 \ln^2(x^4) \cdot \frac{1}{x^4} \cdot (\cancel{4}^{2} x^3)$$

$$= \frac{6 \ln^2(x^4)}{x}$$

$$y = \frac{\sqrt{\ln(x^2-1)}}{4} \qquad y = \frac{1}{4} \left[\ln(x^2-1)\right]^{\frac{1}{2}}$$

$$y' = \frac{1}{4} \cdot \frac{1}{\cancel{2}} \left[\ln(x^2-1)\right]^{\frac{1}{2}-1} \cdot \frac{1}{x^2-1} \cdot (\cancel{2}x)$$

$$y = \frac{x}{4} \left[\ln(x^2-1)\right]^{-\frac{1}{2}} \cdot \frac{1}{x^2-1}$$

$$= \frac{x}{4 \cdot \sqrt{\ln(x^2-1)}\,(x^2-1)}$$

esame (3c) $\quad y = \left[\ln(3x)\right]^4 \qquad y' = 4\left[\ln(3x)\right]^3 \cdot \frac{1}{\cancel{3}x} \cdot \cancel{3} = \frac{4}{x}\left[\ln(3x)\right]^3$

esame (6c) $\quad y = \frac{\ln(1+x)}{1+x} \qquad y' = \frac{\frac{1}{\cancel{1+x}} \cdot \cancel{(1+x)} - \ln(1+x) \cdot 1}{(1+x)^2}$

$$= \frac{1 - \ln(1+x)}{(1+x)^2}$$

esame (5c) $\quad y = \frac{1+x}{\ln(1+x)} \qquad y' = \frac{1 \cdot \ln(1+x) - \cancel{(1+x)} \cdot \frac{1}{\cancel{1+x}} \cdot 1}{\left[\ln(1+x)\right]^2}$

$$= \frac{\ln(1+x) - 1}{\left[\ln(1+x)\right]^2}$$

$$y = x^4 \ln^2 x \qquad y' = 4x^3 \ln^2 x + x^{\cancel{4}\,3} \cdot 2 \ln x \cdot \frac{1}{\cancel{x}} =$$

$$= 4x^3 \ln^2 x + 2x^3 \ln x =$$

$$= 2x^3 \ln x \,(2 \ln x + 1)$$

$$x \qquad y = \ln\left(\frac{1-x}{1+2x}\right) \qquad y' = \frac{1}{\frac{1-x}{1+2x}} \cdot \frac{-1(1+2x) - (1-x)(2x)}{(1+2x)^2}$$

$$= \frac{1}{\frac{1-x}{\cancel{1+2x}}} \cdot \frac{-1-2x-2x+2x^2}{(1+2x)^{\cancel{2}}}$$

$$= \frac{1}{1-x} \cdot (2x^2-4x-1) = \frac{2x^2-4x-1}{1-x}$$

$y = a^{f(x)} \qquad y' = a^{f(x)} \cdot \ln a \cdot f'(x)$

$y = e^{f(x)} \qquad y' = e^{f(x)} \cdot f'(x)$

$y = e^{x^2-1} \qquad y' = e^{x^2-1}\ (2x) = 2x\, e^{x^2-1}$

$y = e^{-x} \qquad y' = e^{-x}\ (-1) = -e^{-x}$

$y = \frac{e^{\frac{x}{4}}}{3} \qquad y = \frac{1}{3}\, e^{\frac{1}{4}x} \qquad y' = \frac{1}{3}\, e^{\frac{1}{4}x} \cdot \frac{1}{4} = \frac{1}{12}\, e^{\frac{1}{4}x}$

$y = e^{\frac{1}{x}} \qquad y = e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)$

$= -\frac{e^{\frac{1}{x}}}{x^2}$

$\left[\, y = \frac{1}{x} \quad y = x^{-1} \Rightarrow y' = -1x^{-2} = \frac{-1}{x^2}\right.$

$y = e^{\sqrt{x}} \qquad y = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$

$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

$\left[\, y = \sqrt{x} \quad y = x^{\frac{1}{2}} \Rightarrow y' = \frac{1}{2}x^{\frac{1}{2}-1} = \right.$

$= \frac{1}{2\sqrt{x}}$

$y = \frac{e^{2x}}{3x} \qquad y' = \frac{e^{2x} \cdot 2 \cdot 3x - e^{2x} \cdot 3}{9x^2}$

$= \frac{6x\, e^{2x} - 3\, e^{2x}}{9x^2} = \frac{\cancel{3}e^{2x}\,(2x-1)}{\cancel{9}_3 x^2}$

$y = x^4\, e^{1-x} \qquad y' = 4x^3\, e^{1-x} - x^4 \cdot e^{1-x}\ (-1)$

$= 4x^3\, e^{1-x} + x^4\, e^{1-x} =$

$= x^3\, e^{1-x}\ (4-x)$

$y = e^{\frac{2x}{1-x}} \qquad y' = e^{\frac{2x}{1-x}} \cdot \frac{2(1-x) - 2x(-1)}{(1-x)^2}$

$= e^{\frac{2x}{1-x}} \cdot \frac{2 - \cancel{2x} + \cancel{2x}}{(1-x)^2}$

$= \frac{2}{(1-x)^2} \cdot e^{\frac{2x}{1-x}}$