DERIVATE

Regole delle funzioni

f. COSTANTE $y = c$ $\quad y' = 0$

f. POTENZA $y = x^n$ $\quad y' = nx^{n-1}$

$y = x^3$ $\quad y' = 3x^{3-1}$

$y = \frac{1}{x^3} = x^{-3}$ $\quad y' = -3x^{-3-1}$

$y = \sqrt[3]{x^2} = x^{\frac{2}{3}}$ $\quad y' = \frac{2}{3}x^{\frac{2}{3}-1}$

$y = \frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$ $\quad y' = -\frac{2}{3}x^{-\frac{2}{3}-1}$

da cui si deduce

$y = x$ $\quad y' = 1$

$y = \sqrt{x}$ $\quad y' = \frac{1}{2\sqrt{x}}$

f. ESPONENZIALE

$y = a^x$ $\quad y' = a^x \cdot \ln a$

$y = e^x$ $\quad y' = e^x$

f. LOGARITMO

$y = \log_a x$ $\quad y' = \frac{1}{x \cdot \ln a}$

$y = \ln x$ $\quad y' = \frac{1}{x}$

f. GONIOMETRICHE

$y = \operatorname{sen} x$ $\quad y' = \cos x$

$y = \cos x$ $\quad y' = -\cos x$

$y = \operatorname{tg} x$ $\quad y' = 1 + \operatorname{tg}^2 x \overset{!}{=} \frac{1}{\cos^2 x}$

$y = \operatorname{cotg} x$ $\quad y' = -1 - \operatorname{cotg}^2 x \overset{!}{=} -\frac{1}{\operatorname{sen}^2 x}$

$y = \operatorname{arctg} x$ $\quad y' = \frac{1}{1+x^2}$

$y = \operatorname{arccotg}$ $\quad y' = -\frac{1}{1+x^2}$

$y = \operatorname{arcsen} x$ $\quad y' = \frac{1}{\sqrt{1-x^2}}$

$y = \arccos x$ $\quad y' = -\frac{1}{\sqrt{1-x^2}}$

Regole delle operazioni

SOMMA / DIFFERENZA

- $y = f(x) \pm g(x)$ $\quad y' = f'(x) \pm g'(x)$

PRODOTTO CON COSTANTE

- $y = k \cdot f(x)$ $\quad y' = k \cdot f'(x)$

PRODOTTO TRA FUNZIONI

- $y = f(x) \cdot g(x)$ $\quad y' = f'(x) \cdot g(x) + f(x) g'(x)$

QUOZIENTE

- $y = \frac{f(x)}{g(x)}$ $\quad y' = \frac{f'(x) g(x) - f(x) g'(x)}{[g(x)]^2}$

Funzione composta

$y = f(\underbrace{g(x)}_{\text{VAR}})$ $\quad y' = f'(\underbrace{g(x)}_{\text{Variabile riscritta}}) \cdot \underbrace{g'(x)}_{\text{Variabile derivata}}$

DERIVATE

$$\boxed{f(x) = x^n \qquad f'(x) = n\, x^{n-1}}$$

$y = x^5 \qquad y' = 5x^4$

$y = x^1 \qquad y' = 1 \cdot \underbrace{x^0}_{=1} = 1$

$y = x \qquad y' = 1$

$y = \sqrt[5]{x^2} \Rightarrow y = x^{\frac{2}{5}} \qquad y' = \frac{2}{5} \cdot x^{\frac{2}{5}-1} = \frac{2}{5} \cdot x^{-\frac{3}{5}} = \frac{2}{3} \cdot \frac{1}{\sqrt[5]{x^3}}$

$y = \sqrt[3]{x} \Rightarrow y = x^{\frac{1}{3}} \qquad y' = \frac{1}{3} \cdot x^{\frac{1}{3}-1} = \frac{1}{3} \cdot x^{-\frac{2}{3}} = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}$

$y = \sqrt{x} \Rightarrow y = x^{\frac{1}{2}} \qquad y' = \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2} \frac{1}{\sqrt{x}}$

$y = \sqrt{x} \qquad y' = \frac{1}{2\sqrt{x}}$

$y = \frac{1}{x^3} \Rightarrow y = x^{-3} \qquad y' = -3 \cdot x^{-4} = -3 \cdot \frac{1}{x^4} = -\frac{3}{x^4}$

$y = \frac{1}{x^2} \Rightarrow y = x^{-2} \qquad y' = -2 \cdot x^{-3} = -2 \cdot \frac{1}{x^3} = -\frac{2}{x^3}$

$y = \frac{1}{x} \Rightarrow y = x^{-1} \qquad y' = -1 \cdot x^{-2} = -1 \cdot \frac{1}{x^2} = -\frac{1}{x^2}$

$y = k\, f(x) \qquad y' = k\, f'(x)$

$y = \frac{3}{2} x^4 \qquad y' = \frac{3}{\cancel{2}} \cdot \cancel{4}^{2} \cdot x^3 = 6 \cdot x^3$

$y = \frac{x^2}{3} \qquad y = \frac{1}{3} \cdot x^2 \qquad y' = \frac{1}{3} \cdot 2 \cdot x = \frac{2}{3} x$

$y = \frac{\sqrt{x}}{4} \qquad y = \frac{1}{4} \cdot x^{\frac{1}{2}} \qquad y' = \frac{1}{4} \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{8} x^{-\frac{1}{2}} = \frac{1}{8} \cdot \frac{1}{\sqrt{x}}$

$y = \frac{1}{2x^3} \qquad y = \frac{1}{2} \cdot x^{-3} \qquad y' = \frac{1}{2} \cdot (-3) \cdot x^{-4} = -\frac{3}{2} \cdot \frac{1}{x^4} = -\frac{3}{2x^4}$

$y = 5 \qquad y = 5 \cdot x^0 \qquad y' = 5 \cdot 0 \cdot x^{-1} = 0 \qquad y = c \qquad y' = 0$

$y = f(x) \pm g(x) \qquad y' = f'(x) \pm g'(x)$

- $y = \frac{x^4}{2} - \frac{\sqrt[3]{x}}{6} + \frac{2}{3x} + 3$

$y = \frac{1}{2}x^4 - \frac{1}{6}\cdot x^{\frac{1}{3}} + \frac{2}{3}\cdot x^{-1} + 3$

$y' = \frac{1}{2}\cdot \cancel{4}^{2} x^3 - \frac{1}{6}\cdot\frac{1}{3}x^{\frac{1}{3}-1} + \frac{2}{3}\cdot(-1)\,x^{-2} + 0$

$y' = 2x^3 - \frac{1}{18}x^{-\frac{2}{3}} - \frac{2}{3}\cdot\frac{1}{x^2}$

$y' = 2x^3 - \frac{1}{18\sqrt[3]{x^2}} - \frac{2}{3x^2}$

- $y = \frac{2x^3 + \sqrt{x} - 4x}{3}$

$y = \frac{2x^3}{3} + \frac{\sqrt{x}}{3} - \frac{4}{3}x \qquad y = \frac{2}{3}x^3 + \frac{1}{2}x^{\frac{1}{2}} - \frac{4}{3}x$

$y' = \frac{2}{\cancel{3}}\cdot\cancel{3}x^2 + \frac{1}{2}\cdot\frac{1}{2}\cdot x^{\frac{1}{2}-1} - \frac{4}{3}\cdot(1)$

$y' = 2x^2 + \frac{1}{4}x^{-\frac{1}{2}} - \frac{4}{3}$

$y' = 2x^3 + \frac{1}{4\sqrt{x}} - \frac{4}{3}$

- $y = \frac{x^3 - 2x + 3}{x^2}$

$y = \frac{\cancel{x^3}}{\cancel{x^2}} - \frac{2\cancel{x}}{x^{\cancel{2}}} + \frac{3}{x^2} \qquad y = x - \frac{2}{x} + \frac{3}{x^2}$

$y = x - 2\cdot x^{-1} + 3\cdot x^{-2}$

$y' = 1 - 2(-1)\,x^{-2} + 3(-2)\cdot x^{-3}$

$y' = 1 + \frac{2}{x^2} - \frac{6}{x^3}$

$y = a^x \qquad y' = a^x \cdot \ln a$

$y = e^x \qquad y' = e^x \underbrace{\ln e}_{=1} = e^x = 1$

$\text{con } x > 0$

$y = \log_a x \qquad y' = \dfrac{1}{x \cdot \ln a} \qquad \left[y' = \dfrac{1}{x} \cdot \log_a e \right]$

$y = \ln x \qquad y' = \dfrac{1}{x \underbrace{\ln e}_{=1}} = \dfrac{1}{x} \qquad \text{con } x > 0$

- $y = \dfrac{2^x}{3} \qquad y = \dfrac{1}{3} \cdot 2^x \qquad y' = \dfrac{1}{3} \cdot 2^x \cdot \ln 2$
- $y = \dfrac{e^x}{2} \qquad y = \dfrac{1}{2} \cdot e^x \qquad y' = \dfrac{1}{2} e^x$
- $y = \dfrac{\log_3 x}{4} \qquad y = \dfrac{1}{4} \log_3 x \qquad y' = \dfrac{1}{4} \cdot \dfrac{1}{x \cdot \ln 4} = \dfrac{1}{4x \ln 4}$

$y = \dfrac{2 \ln x}{3} \qquad y = \dfrac{2}{3} \ln x \qquad y' = \dfrac{2}{3} \cdot \dfrac{1}{x} = \dfrac{2}{3x}$

$y = 3 \cdot 4^x - \dfrac{\ln x}{2} + \dfrac{4e^x}{5} - 2x \qquad y = 3 \cdot 4^x - \dfrac{1}{2} \ln x + \dfrac{4}{5} e^x - 2x$

$y' = 3 \cdot 4^x \ln 4 - \dfrac{1}{2} \cdot \dfrac{1}{x} + \dfrac{4}{5} e^x - 2$

$y = \dfrac{5 \cdot \log_2 x}{4} - 4 \cdot \ln x + \dfrac{2}{x} \qquad y = \dfrac{5}{4} \log_2 x - 4 \ln x + 2 \cdot x^{-1}$

$y' = \dfrac{5}{4} \cdot \dfrac{1}{x \ln 2} - 4 \cdot \dfrac{1}{x} + 2(-1) \cdot x^{-2}$

$y' = \dfrac{5}{4x \ln 2} - \dfrac{4}{x} - \dfrac{2}{x^2}$

$y = \operatorname{sen} x \qquad y' = \cos x$

$y = \cos x \qquad y' = -\operatorname{sen} x$

$y = \dfrac{\cos x}{2} - \dfrac{4 \operatorname{sen} x}{5} + \pi \qquad y = \dfrac{1}{2} \cos x - \dfrac{4}{5} \operatorname{sen} x + \pi$

$y' = \dfrac{1}{2} (-\operatorname{sen} x) - \dfrac{4}{5} \cos x + 0$

$y' = -\dfrac{1}{2} \operatorname{sen} x - \dfrac{4}{5} \cos x$

$y = \sqrt{2} - \dfrac{2 \cos x}{3} \qquad y' = 0 - \dfrac{2}{3} \cdot (-\operatorname{sen} x) = +\dfrac{2}{3} \operatorname{sen} x$

$$y = f(x) \cdot g(x) \qquad y' = f'(x)\, g(x) + f(x)\, g'(x)$$

$$y = \frac{f(x)}{g(x)} \qquad y' = \frac{f'(x)\, g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$y = e^x (x^2 - x) \qquad y' = e^x (x^2 - x) + e^x (2x - 1) =$$

$$= e^x (x^2 - x + 2x - 1)$$

$$= e^x (x^2 + x - 1)$$

$x^{\frac{1}{2}}$

$$y = \sqrt{x}\,(1 - x) \qquad y' = \frac{1}{2\sqrt{x}}(1 - x) + \sqrt{x}\,(-1) =$$

$$= \frac{1-x}{2\sqrt{x}} - \sqrt{x} = \frac{1 - x - 2x}{2\sqrt{x}} = \frac{1-3x}{2\sqrt{x}}$$

$$y = \frac{x^2}{1-x} \qquad y' = \frac{2x(1-x) - x^2(-1)}{(1-x)^2} =$$

$$= \frac{2x \cancel{- x^2} \cancel{+ x^2}}{(1-x)^2} = \frac{2x}{(1-x)^2}$$

$$y = \frac{x^4 e^x}{2} \qquad y = \underbrace{\frac{1}{2} x^4}\cdot \underbrace{e^x}$$

$$y' = \frac{1}{2} \cdot \cancel{4}^{2} x^3 \cdot e^x + \frac{1}{2} x^4 \cdot e^x =$$

$$= 2x^3 e^x + \frac{1}{2} x^4 e^x = x^3 e^x \left(2 + \frac{1}{2} x\right)$$

$$y = \frac{e^x \ln x}{3} \qquad y = \underbrace{\frac{1}{3} e^x} \cdot \underbrace{\ln x}$$

$$y' = \frac{1}{3} e^x \cdot \ln x + \frac{1}{3} e^x \cdot \frac{1}{x} =$$

$$= \frac{1}{3} e^x \left(\ln x + \frac{1}{x}\right)$$

$$y = \frac{e^x (1 - \cos x)}{2} \qquad y = \underbrace{\frac{1}{2} e^x} (1 - \cos x)$$

$$y' = \frac{1}{2} e^x (1 - \cos x) + \frac{1}{2} e^x (-(-\operatorname{sen} x))$$

$$= \frac{1}{2} e^x (1 - \cos x) + \frac{1}{2} e^x \operatorname{sen} x$$

$$= \frac{1}{2} e^x (1 - \cos x + \operatorname{sen} x)$$

$y = \frac{1-x}{x^2-4}$ $\quad y' = \frac{-1(x^2-4)-(1-x)2x}{(x^2-4)^2}$

$y' = \frac{-x^2+4-2x+2x^2}{(x^2-4)^2} =$

$= \frac{x^2-2x+4}{(x^2-4)^2}$

$y = \frac{x}{\ln x}$ $\quad y' = \frac{1\cdot\ln x - \cancel{x}\cdot\frac{1}{\cancel{x}}}{\ln^2 x} = \frac{\ln x - 1}{\ln^2 x}$

$y = \frac{1-x^2}{e^x}$ $\quad y' = \frac{-2x\cdot e^x-(1-x^2)e^x}{(e^x)^2} = \frac{\cancel{e^x}(-2x-1+x^2)}{(e^x)^{\cancel{2}}}$

$= \frac{x^2-2x-1}{e^x}$

Esame (6c) $y = -x^2\cdot\sqrt[4]{x}$ $\quad y = \underbrace{-x^2}\cdot\underbrace{x^{\frac{1}{4}}}$ $\quad y' = -2x\cdot x^{\frac{1}{4}} + (-x^2)\cdot\frac{1}{4}x^{\frac{1}{4}-1}$

$= -2x\sqrt[4]{x} - \frac{x^2}{4}x^{-\frac{3}{4}} =$

$= -2x\sqrt[4]{x} - \frac{x^2}{4\sqrt[4]{x^3}} =$

$= \frac{-8x\sqrt[4]{x^4}-x^2}{4\sqrt[4]{x^3}} = \frac{-9x^2}{4\sqrt[4]{x^3}}$

$= -\frac{9}{4}x^{2-\frac{3}{4}} = -\frac{9}{4}x^{\frac{5}{4}} =$

$= -\frac{9}{4}\sqrt[4]{x^5}$

$y = -x^2\cdot\sqrt[4]{x}$ $\quad y = -x^2\cdot x^{\frac{1}{4}} = -x^{2+\frac{1}{4}} = -x^{\frac{9}{4}}$

$y' = -\frac{9}{4}x^{\frac{9}{4}-1} = -\frac{9}{4}x^{\frac{5}{4}} = -\frac{9}{4}\sqrt[4]{x^5}$