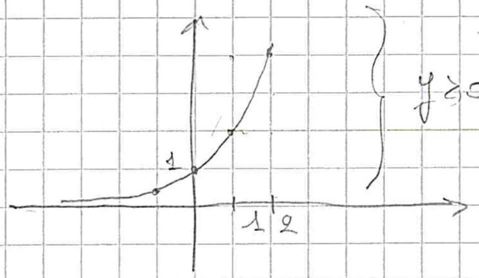
FUNZIONE ESPONENZIALE

$y = f(x) = a^x$, con $a > 0$ $f: \mathbb{R} \to]0, +\infty[$

$\boxed{a > 1}$

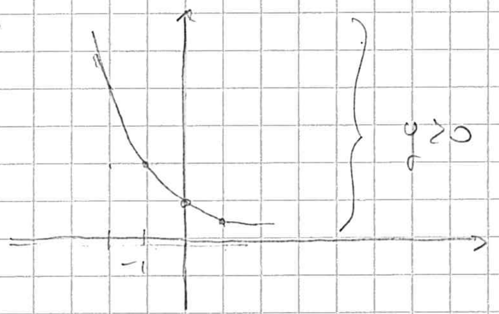


$y \geq 0$

$y = 2^x$

x	y
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
-1	$2^{-1} = +\frac{1}{2}$

$\boxed{0 < a < 1}$



$y \geq 0$

$y = \left(\frac{1}{2}\right)^x$

x	y
0	$\left(\frac{1}{2}\right)^0 = 1$
1	$\left(\frac{1}{2}\right)^1 = \frac{1}{2}$
-1	$\left(\frac{1}{2}\right)^{-1} = 2$
-2	$\left(\frac{1}{2}\right)^{-2} = 4$

PROPRIETÀ

$a^x > 0 \quad \forall x \in \mathbb{R}$ da cui si ricava

$a^x = 0$ impossibile

$a^x \neq 0$ sempre, $\forall x \in \mathbb{R}$

$a^x < 0$ impossibile

$a^x \leq 0$ impossibile

$a^x \geq 0$ $\forall x \in \mathbb{R}$ (perché è $a^x > 0 \ \forall x \in \mathbb{R}$)

$a^0 = 1$

$a^1 = a$

STESSA BASE

$a^x \cdot a^y = a^{x+y}$ $a^x + a^y =$ STOP

$\frac{a^x}{a^y} = a^{x-y}$ $a^x - a^y =$ STOP

POTENZA DI POTENZA

$(a^x)^y = a^{x \cdot y}$

ESPONENTE NEGATIVO

$a^{-x} = \frac{1}{a^x}$

$\left(\frac{a}{b}\right)^{-x} = \left(\frac{b}{a}\right)^x$

ESPONENTE FRAZIONARIO

$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Esempi delle proprietà con esponente negativo ed esponente frazione

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$

$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

$$\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$$

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4}$$

$$\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^{1} = 2$$

$$\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^{2} = 4$$

$$\sqrt[2]{2^1} = 2^{\frac{1}{2}}$$

$$\sqrt[4]{3^1} = 3^{\frac{1}{4}}$$

$$\sqrt[4]{2^3} = 2^{\frac{3}{4}}$$

$$2^{\frac{1}{3}} = \sqrt[3]{2^1}$$

$$2^{\frac{4}{3}} = \sqrt[3]{2^4}$$

$$2^{-\frac{1}{3}} = \frac{1}{2^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{2}}$$

$$2^{-\frac{3}{4}} = \frac{1}{2^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{2^3}}$$

EQUAZIONI ESPONENZIALI

Teor: Se $a^{x_1} = a^{x_2} \iff x_1 = x_2$ (perché $f(x) = a^x$ è funzione iniettiva cioè $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$)

Es 1

- $2^x = 32 \Rightarrow 2^x = 2^5 \Rightarrow x = 5$

$10^x = 0{,}01 \Rightarrow 10^x = \frac{1}{100} \Rightarrow 10^x = \frac{1}{10^2} \Rightarrow 10^x = 10^{-2}$

$\Rightarrow x = -2$

- $81^{x(x+2)} = 27^{\frac{4}{3}x+5} \Rightarrow (3^4)^{x(x+2)} = (3^3)^{\frac{4}{3}x+5}$

$\Rightarrow 3^{4x(x+2)} = 3^{3\left(\frac{4}{3}x+5\right)}$

$\Rightarrow 4x(x+2) = 3\left(\frac{4}{3}x+5\right)$

$\Rightarrow 4x^2 + 8x = 4x + 15$

$\Rightarrow 4x^2 + 4x - 15 = 0$

$x_{1,2} = \frac{-4 \pm \sqrt{16+240}}{8} = \frac{4 \pm \sqrt{256}}{8}$

$= \frac{-4 \pm 16}{8} = \begin{cases} \frac{-20}{8} = -\frac{5}{2} \\ +\frac{12}{8} = \frac{3}{2} \end{cases}$

- $2^x = 16 \cdot \sqrt{2} \Rightarrow 2^x = 2^4 \cdot 2^{\frac{1}{2}}$

$\Rightarrow 2^x = 2^{4+\frac{1}{2}} \Rightarrow x = 4 + \frac{1}{2} \Rightarrow x = \frac{8+1}{2} \Rightarrow x = \frac{9}{2}$

$4^x = 2 \cdot \sqrt{2} \Rightarrow (2^2)^x = 2 \cdot 2^{\frac{1}{2}} \Rightarrow 2^{2x} = 2^{1+\frac{1}{2}}$

$\Rightarrow 2x = 1 + \frac{1}{2}$

$\Rightarrow \cancel{2} \cdot \frac{4x}{\cancel{2}} = \frac{2+1}{\cancel{2}} \cdot \cancel{2}$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{3}{4}$

$x = \frac{3}{4}$

$$5^x = \frac{\sqrt{5}}{25} \Rightarrow 5^x = \frac{5^{\frac{1}{2}}}{5^2} \rightarrow 5^x = 5^{\frac{1}{2}-2}$$

$$\Rightarrow x = \frac{1}{2} - 2$$

$$x = \frac{1-4}{2}$$

$$x = -\frac{3}{2}$$

- $$27^x = \frac{9 \cdot \sqrt{3}}{\sqrt[4]{3}} \Rightarrow (3^3)^x = \frac{3^2 \cdot 3^{\frac{1}{2}}}{3^{\frac{1}{4}}} \Rightarrow 3^{3x} = 3^{2+\frac{1}{2}-\frac{1}{4}}$$

$$\Rightarrow 3x = 2 + \frac{1}{2} - \frac{1}{4}$$

$$\cancel{4} \cdot \frac{12x}{\cancel{4}} = \frac{8+2-1}{4} \cdot \cancel{4}$$

$$\frac{\cancel{12}x}{\cancel{12}} = \frac{9}{12} \Rightarrow x = \frac{9}{12} \qquad x = \frac{3}{4}$$

- $$8^x \cdot \sqrt{2} = 1 \Rightarrow (2^3)^x \cdot 2^{\frac{1}{2}} = 2^0$$

$$2^{3x+\frac{1}{2}} = 2^0$$

$$3x + \frac{1}{2} = 0$$

$$\cancel{2} \cdot \frac{6x+1}{\cancel{2}} = \frac{0}{\cancel{2}} \cdot \cancel{2}$$

$$6x = -1$$

$$x = -\frac{1}{6}$$

B

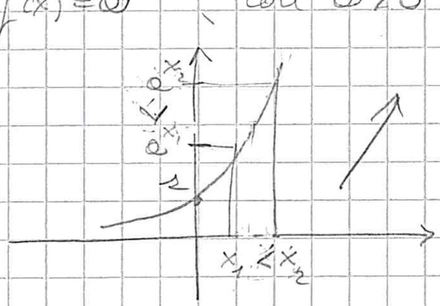
$2^x = 0$ impossible

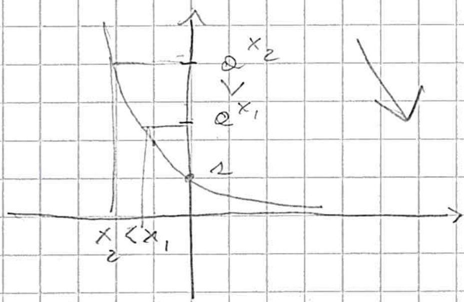
$2^x = -1$ impossible

$2^x = 1 \Rightarrow 2^x = 2^0 \Rightarrow x = 0$

DISEQUAZIONI ESPONENZIALI

$y = f(x) = a^x$ con $a > 0$





Con $a > 1$

Se $a^{x_1} \underset{>}{<} a^{x_2} \Rightarrow x_1 \underset{>}{<} x_2$

(viene mantenuto il verso della disequazione)

"STESSO VERSO"

Con $0 < a < 1$

Se $a^{x_1} \underset{>}{<} a^{x_2} \Rightarrow x_1 \underset{<}{>} x_2$

(viene invertito il verso della disequazione)

"CAMBIO IL VERSO"

- $\sqrt[3]{7^x} > 49 \Rightarrow 7^{\frac{x}{3}} > 7^2 \Rightarrow \cancel{3} \cdot \frac{x}{\cancel{3}} > 2 \cdot 3 \Rightarrow x > 6$

- $4^{3x} < \frac{1}{2} \Rightarrow (2^2)^{3x} < 2^{-1} \Rightarrow 2^{6x} < 2^{-1} \Rightarrow \frac{\cancel{6}x}{\cancel{6}} < -\frac{1}{6}$

  $x < -\frac{1}{6}$

- $\left(\frac{1}{2}\right)^{x+1} > \frac{1}{4} \Rightarrow \left(\frac{1}{2}\right)^{x+1} > \left(\frac{1}{2}\right)^2 \Rightarrow x+1 < 2 \Rightarrow x < 1$

- $\left(\frac{2}{3}\right)^{2x} < \frac{27}{8} \Rightarrow \left(\frac{2}{3}\right)^{2x} < \frac{3^3}{2^3} \Rightarrow \left(\frac{2}{3}\right)^{2x} < \left(\frac{3}{2}\right)^3 \Rightarrow$

  $\Rightarrow \left(\frac{2}{3}\right)^{2x} < \left(\frac{2}{3}\right)^{-3} \Rightarrow \frac{\cancel{2}x}{\cancel{2}} > \frac{-3}{2} \Rightarrow x > -\frac{3}{2}$

- $9^{x-1} > 1 \Rightarrow (3^2)^{x-1} > 3^0 \Rightarrow 3^{2(x-1)} > 3^0$

  $\Rightarrow 2(x-1) > 0 \Rightarrow 2x - 2 > 0$

  $\frac{\cancel{2}x}{\cancel{2}} > \frac{\cancel{2}}{\cancel{2}} \Rightarrow x > 1$

CASI IMPORTANTI

$9^x > 0$ ($\geq$) sempre, $\forall x \in \mathbb{R}$

$9^x < 0$ ($\leq$) impossibile

$9^x > -3$ sempre $\forall x \in \mathbb{R}$

$9^x \geq -3$ sempre, $\forall x \in \mathbb{R}$

$9^x < -3$ ($\leq$) impossibile

$9^x > 3 \Rightarrow (3^2)^x > 3^1 \Rightarrow 3^{2x} > 3^1 \Rightarrow \frac{\cancel{2}x}{\cancel{2}} > \frac{1}{2} \Rightarrow x > \frac{1}{2}$

$9^x < 3 \Rightarrow (3^2)^x < 3^1 \Rightarrow 3^{2x} < 3^1 \Rightarrow \frac{\cancel{2}x}{\cancel{2}} < \frac{1}{2} \Rightarrow x < \frac{1}{2}$

$2^x + 1 > 0 \Rightarrow 2^x > -1$ sempre, $\forall x \in \mathbb{R}$

$2^x + 1 < 0 \Rightarrow 2^x < -1$ impossibile

$2^x - 1 > 0 \Rightarrow 2^x > 1 \Rightarrow 2^x > 2^0 \Rightarrow x > 0$

$2^x - 1 < 0 \Rightarrow 2^x < 1 \Rightarrow 2^x < 2^0 \Rightarrow x < 0$